

# A Zero-Pole Whitening Filter in Adaptive Blind Decision Feedback Equalizers

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**Abstract**—This paper proposes the use of a novel recursive *zero-pole whitening filter (ZPWF)* instead of the *all-pole whitening filter (APWF)* and the *all-zero whitening filter (AZWF)* in the adaptive blind decision feedback equalizer (DFE) system. Both the *APWF* and the *AZWF* are special cases of the proposed *ZPWF*. The rationale for proposing the *ZPWF* may be explained as follows. Any arbitrary non-minimum phase channel may be approximated by the following three different types of channel: the moving-average (MA) channel, the autoregressive (AR) channel and the autoregressive-moving-average (ARMA) channel. The MA channel or the AR channel may be more effectively approximated by the ARMA channel with a much smaller total (numerator and denominator) number of coefficients. From the perspective of channel equalization, the *APWF* and the *AZWF* may merely suit a particular type of channel, while the proposed *ZPWF* may suit most channels in general.

**Keywords**—Adaptive blind equalization, Decision feedback equalizer (DFE), Zero-Pole whitening filter (ZPWF)

## I. INTRODUCTION

Linear equalizers (LEs) cause noise enhancement when they are used to eliminate the residual intersymbol interference (ISI) from the input signal. In order to solve this problem, the decision feedback equalizer (DFE) is often used to replace the LE, and the former has much better error rate performance than the latter especially in severe fading channels. In the trained adaptive DFE, the periodic pseudo-random data sequence has to occur periodically for time-varying situations, which may result in bandwidth loss. Accordingly, a clear advantage of conventional blind adaptive DFE over a trained adaptive DFE in long multipath and dynamic multipath channels has been demonstrated in [1] and the former enables faster acquisition than does the latter. However, the conventional blind adaptive DFE may cause the error propagation phenomenon owing to the sudden change of the channels, and several techniques have been proposed to mitigate the DFE error propagation [2]-[6].

Labat, Macchi and Laot [3] proposed a blind adaptive DFE employing the two-mode scheme (i.e., starting mode and tracking mode). The starting mode of a blind adaptive DFE employs a combination of an infinite impulse response (IIR) all-pole whitening filter (APWF) along with a blind equalizer. Once the eye is open, the equalizer then switches from the starting mode to the tracking mode in which a decision-directed equalizer (DDE) and an adaptive

DFE are employed. With a sudden channel change, the equalizer may switch back to the starting mode in order to avoid the error propagation phenomenon. More recently, Lim, Kennedy and Abhayapala [4] proposed a blind adaptive DFE using finite impulse response (FIR) all-zero whitening filter (AZWF) to replace the IIR APWF in [3]. This work proposes a novel adaptive blind DFE employing the IIR zero-pole whitening filter (ZPWF). The ZPWF based on an autoregressive-moving-average (ARMA) model may be used to replace the APWF based on an autoregressive (AR) model and the AZWF based on a moving-average (MA) model, since the ZPWF can be more economically modeled with both poles and zeros than the AZWF (APWF) with all-zero (all-pole) alone [7]. Therefore, the use of the ZPWF may reduce the complexity of the computation of the adaptive blind DFE in both the starting mode and the tracking mode. It is well known that a long feedback filter in the DFE may be more prone to severe error propagation and therefore constrained feedback weights can improve DFE performance [5], [8]-[9]. The proposed DFE employing the ZPWF, which may approximate a long feedback filter with a smaller number of poles and zeros, may thus mitigate error propagation. Moreover, the ZPWF has more flexibility in terms of implementation than both the APWF and the AZWF because the tap weights exist in both the numerator and the denominator of the ZPWF, whereas the tap weights of the APWF (AZWF) merely exist in the denominator (numerator). Although filters with both poles and zeros are known to be widely used in speech coding, system identification and channel equalization [10]-[16], this paper focuses on applying the ZPWF as the most general whitening filter to equalize various types of channel such as the MA channel, AR channel and ARMA channel.

## II. SYSTEM MODEL AND THE PROPOSED ZPWF

### A. System Model

The autoregressive-moving-average (ARMA) channel in the baseband model of a communication system shown in Fig. 1 may be denoted by [14], [17]

$$C(z) = \frac{C_z(z)}{C_p(z)} = \frac{\sum_{i=0}^{M_z} c_{z,i} \cdot z^{-i}}{1 + \sum_{j=1}^{M_p} c_{p,j} \cdot z^{-j}} \quad (1)$$

$C(z)$  in (1) is the general form of a causal linear recursive discrete-time channel model since it may be reduced to either a FIR channel such as a MA channel (when  $M_p = 0$ ) or an IIR channel such as an AR channel (when  $M_z = 0$  ( $c_{z,0} \neq 0$ )). In (1),  $c_{z,i}$ ,  $i = 0, \dots, M_z$ , denotes the channel coefficients of the numerator in which  $M_z$  represents its order;  $c_{p,j}$ ,  $j = 1, \dots, M_p$ , denotes the channel coefficients of the denominator in which  $M_p$  represents its order. The transmitted signal,  $s(n)$ , is assumed to be independent and identically distributed (iid) random variables with variance  $\sigma_s^2$  as depicted in Fig. 1. The equalizer input,  $r(n)$ , can then be expressed as

$$r(n) = \sum_{i=0}^{M_z} c_{z,i} \cdot s(n-i) + \sum_{j=1}^{M_p} c_{p,j} \cdot l(n-j) + w(n) \quad (2)$$

in which  $l(n)$  is the channel output;  $w(n)$  is additive white Gaussian noise (AWGN) with variance  $\sigma_w^2$ , and  $F(z)$  is the equalizer whose output,  $y(n)$ , is the input of the decision device  $Q\{\cdot\}$ . The output of the decision device then generates  $\hat{s}(n)$ , which is the estimated  $s(n)$ .

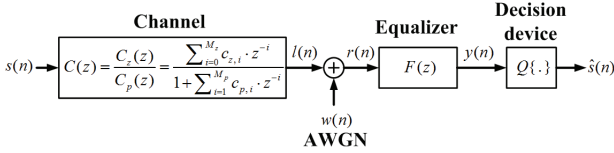


Fig. 1. Baseband model of a communication system.

When the mean-squared error (MSE) criterion is used, the equalizer of infinite length,  $F(z)$ , is adjusted to minimize the cost function

$$J = E\{|e(n)|^2\} \quad (3)$$

where  $e(n) = s(n-\delta) - y(n)$  is the estimation error for a particular choice of delay ( $\delta$ ). According to [8], [18], [19], the minimum mean-squared error linear equalizer (MMSE LE) can be derived to be

$$F_{MMSE}(z) = z^{-\delta} \frac{\sigma_s^2 \cdot C^*(1/z^*)}{\sigma_s^2 \cdot C(z) \cdot C^*(1/z^*) + \sigma_w^2} \quad (4)$$

$$= z^{-\delta} \frac{\sigma_s^2 \cdot C^*(1/z^*)}{S_0 \cdot G(z) \cdot G^*(1/z^*)}$$

where  $G(z)$  is constrained to be causal and minimum phase with the same number of roots as that of  $C(z)$ ;  $S_0$  denotes a real positive number that depends on  $\sigma_s^2$ ,  $\sigma_w^2$  and  $C(z)$ . For the noiseless case ( $\sigma_w^2 = 0$ ), the optimal MMSE LE in (4) can be reduced to the zero forcing (ZF) equalizer such that

$$F_{ZF}(z) = z^{-\delta} \cdot \frac{1}{C(z)} \quad (5)$$

$C(z)$  in (1) may be further extended to a *non-minimum phase ARMA channel* in which some zeros of  $C(z)$  are outside the unit circle such that

$$C(z) = \eta \cdot \frac{\prod_{j=1}^{M_{z1}} (1 - c_{zi,j} \cdot z^{-1}) \cdot \prod_{j=1}^{M_{z2}} (1 - c_{zo,j} \cdot z^{-1})}{\prod_{j=1}^{M_p} (1 - c_{pi,j} \cdot z^{-1})} \quad (6)$$

where  $\eta$  is a complex attenuation factor,  $M_z = M_{z1} + M_{z2}$ ;  $c_{zi,j}$  and  $c_{zo,j}$  denote the zeros inside and outside unit circle, respectively, on complex plane such that  $|c_{zi,j}| \leq 1$  and  $|c_{zo,j}| > 1$ ;  $c_{pi}$  denotes the pole inside the unit circle on complex plane such that  $|c_{pi,j}| \leq 1$ . Equation (6) can be decomposed into amplitude distortion and phase distortion such that

$$C(z) = \eta \cdot \underbrace{\left[ \frac{\prod_{j=1}^{M_{z1}} (1 - c_{zi,j} \cdot z^{-1}) \cdot \prod_{j=1}^{M_{z2}} (c_{zo,j}^* \cdot z^{-1})}{\prod_{j=1}^{M_p} (1 - c_{pi,j} \cdot z^{-1})} \right]}_{\text{Amplitude Distortion}} \quad (7)$$

$$\cdot \underbrace{\left[ \frac{\prod_{j=1}^{M_{z2}} (1 - c_{zo,j} \cdot z^{-1})}{\prod_{j=1}^{M_{z2}} (c_{zo,j}^* \cdot z^{-1})} \right]}_{\text{Phase Distortion}} = \eta \cdot C_{AD}(z) \cdot C_{PD}(z)$$

where  $C_{AD}(z)$  and  $C_{PD}(z)$  denote the parts of amplitude distortion and phase distortion, respectively. Substituting (7) into (5) yields the following ZF equalizer

$$F_{ZF}(z) = \frac{1}{\eta} \cdot \underbrace{\left[ \frac{\prod_{j=1}^{M_p} (1 - c_{pi,j} \cdot z^{-1})}{\prod_{j=1}^{M_{z1}} (1 - c_{zi,j} \cdot z^{-1}) \cdot \prod_{j=1}^{M_{z2}} (c_{zo,j}^* \cdot z^{-1})} \right]}_{\text{Amplitude Equalizer (AE)}} \quad (8)$$

$$\cdot \underbrace{\left[ \frac{z^{-\delta} \cdot \prod_{j=1}^{M_{z2}} (c_{zo,j} \cdot z^{-1})}{\prod_{j=1}^{M_{z2}} (1 - c_{zo,j} \cdot z^{-1})} \right]}_{\text{Blind Equalizer (BE)}}$$

The complex attenuation factor  $\eta$ , amplitude distortion, and phase distortion will be compensated by gain control (GC), amplitude equalizer (AE), and blind equalizer (BE), respectively [3]. This paper focuses on the realization of the AE, which is essentially a whitening filter.

## B. Motivation for the Proposed ZPWF

As demonstrated in (1), the ARMA channel may be expressed in terms of the poles and the zeros, which may be cancelled out, respectively, by the zeros and the poles of the AE in order to compensate for the amplitude distortion owing to channels. A novel structure of the AE, referred to herein as the zero-pole whitening filter (ZPWF) depicted in Fig. 2 is proposed. The ZPWF may be expressed as

$$A(z) = \frac{1 + A_z(z)}{1 + A_p(z)} = \frac{1 + \sum_{i=1}^{N_{az}} a_{z,i}(n) \cdot z^{-i}}{1 + \sum_{j=1}^{N_{ap}} a_{p,j}(n) \cdot z^{-j}} \quad (9)$$

where  $a_{z,i}(n)$ , for  $i = 1 \sim N_{az}$ , and  $a_{p,j}(n)$ , for  $j = 1 \sim N_{ap}$ , are tap weights of the ZPWF. When  $N_{az} > 0$  and  $N_{ap} = 0$ , then  $A(z) = 1 + A_z(z)$  is reduced to the all-zero whitening filter (AZWF) proposed in [4], which is an FIR filter and it most suits to compensate for

the AR channel when  $N_{az} = M_p$ . When  $N_{az} = 0$  and  $N_{ap} > 0$ , then  $A(z) = 1/\sqrt{1 + A_p(z)}$  is reduced to the all-pole whitening filter (APWF) proposed in [3], which is an IIR filter and it most suits to compensate for the MA channel when  $N_{ap} = M_z$ .

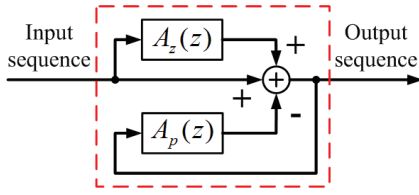


Fig. 2. The proposed ZPWF.

The motivation for applying the ZPWF in the adaptive blind DFE may be demonstrated by the following example. Consider the ARMA minimum-phase channel proposed in [17] with the following transfer function in which all the zeros and poles are inside the unit circle

$$C'(z) = \frac{1 + 0.6 \cdot z^{-1} - 0.3937z^{-3}}{1 - 0.6561 \cdot z^{-4}} \quad (10)$$

$$= \frac{(1 - (-0.589 + 0.578j)z^{-1})(1 - (-0.589 - 0.578j)z^{-1})(1 - 0.578z^{-1})}{(1 - 0.9z^{-1})(1 + 0.9z^{-1})(1 - (0.9j)z^{-1})(1 + (0.9j)z^{-1})}$$

The amplitude distortion  $C_{AD}(z)$  in (7) is therefore identical to  $C'(z)$ , which may be fully compensated by using the following ZPWF with only eight tap weights whose transfer function may be expressed as

$$F_{ZF}(z) = \frac{1}{C'(z)} = \frac{1 - 0.6561 \cdot z^{-4}}{1 + 0.6 \cdot z^{-1} - 0.3937z^{-3}} \quad (11)$$

However, both the APWF and the AZWF would need at least 20 tap weights to be able to compensate for the distortion caused by the ARMA channel in (10) as depicted in Figs. 3(a) and 3(b). This result demonstrates that the ZPWF may indeed be more economically modeled with both poles and zeros than the AZWF (APWF) with all-zero (all-pole) alone to compensate for the distortion caused by the ARMA channel. Moreover, the ZPWF has one additional degree of freedom and more flexibility than either the APWF or the AZWF does because the tap weights exist in both the numerator and the denominator of the ZPWF, whereas the tap weights of the APWF (AZWF) merely exist in the denominator (numerator).

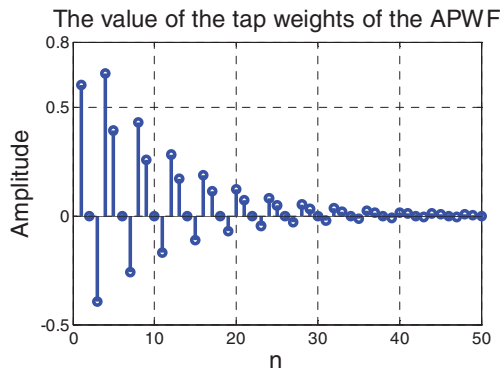


Fig. 3(a). The values of the tap weights of the corresponding APWF based on (11) to compensate for the ARMA channel in (10).

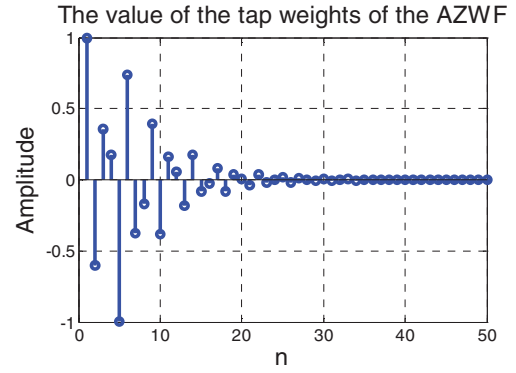


Fig. 3(b). The values of the tap weights of the corresponding AZWF based on (11) to compensate for the ARMA channel in (10).

### III. ADAPTIVE BLIND DFE USING ZERO-POLE WHITENING FILTER (ZPWF)

#### A. Adaptive Blind DFE using ZPWF in Starting Mode

It is well known that error propagation may arise in the conventional DFE. Labat, Macchi, and Laot [3] therefore proposed an adaptive blind DFE in which the use of an APWF as the whitening filter along with a blind equalizer by employing a two-mode scheme (i.e., starting mode and tracking mode). The rationale behind the use of the two-mode scheme is that in the starting mode, the adaptive blind DFE is recursively and blindly adapted by criteria leading to a solution approaching the MMSE solution. Once the adaptive blind DFE converges in the starting mode, it switches to the tracking mode and the adaptive blind DFE becomes the conventional DFE such that error propagation may well be prevented [20]. Lim, Kennedy and Abhayapala [4] then proposed the use of the AZWF to replace the APWF in the adaptive blind DFE. This paper proposes the use of the ZPWF instead of the APWF and the AZWF in the adaptive blind DFE, respectively. The starting mode of the adaptive blind DFE can be shown in Fig. 4 and the algorithms corresponding to the three stages can be described as follows.

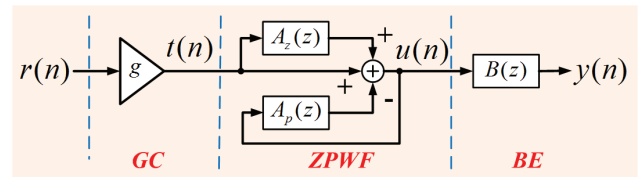


Fig. 4. The adaptive blind DFE using the ZPWF in the starting mode.

(a) Gain Control (GC): The GC may be recursively updated by using  $G(n+1) = G(n) + \mu_g \cdot [\sigma_s^2 - |u(n)|^2]$  such that its output may be computed by  $t(n) = g(n) \cdot r(n)$ , where  $G(0) = 1$ ,  $g(n) = \sqrt{|G(n)|}$  and  $\mu_g$  is the step-size parameter.

(b) Zero-Pole Whitening Filter (ZPWF): The ZPWF may be recursively updated by minimizing  $J_{MOE} = E\{|u(n)|^2\}$ , which is referred to as the minimum

output energy (MOE) algorithm [18], [19], [21], such that the output of the ZPWF can be computed by  $u(n) = t(n) + \sum_{i=1}^{N_{az}} a_{z,i}^*(n) \cdot t(n-i) - \sum_{j=1}^{N_{ap}} a_{p,j}^*(n) \cdot u(n-j)$ . Adaptive implementation of minimizing the cost function,  $J_{MOE}$ , may be realized by the stochastic gradient descent (SGD) method such that

$$a_{z,i}(n+1) = a_{z,i}(n) - \mu_{w1} \cdot \nabla J_{MOE} = a_{z,i}(n) - \mu_{w1} \cdot \frac{\partial J_{MOE}}{\partial a_{z,i}^*(n)},$$

for  $i = 1 \sim N_{az}$  (12)

$$a_{p,j}(n+1) = a_{p,j}(n) - \mu_{w1} \cdot \nabla J_{MOE} = a_{p,j}(n) - \mu_{w1} \cdot \frac{\partial J_{MOE}}{\partial a_{p,j}^*(n)},$$

for  $j = 1 \sim N_{ap}$  (13)

where  $\mu_{w1}$  is step-size parameter and  $\frac{\partial J_{MOE}}{\partial a_{z,i}^*(n)} = u^*(n) \cdot t(n-i)$ , for  $i = 1 \sim N_{az}$ . Since the

gradient in  $\frac{\partial J_{MOE}}{\partial a_{p,j}^*(n)}$  may lead to cumbersome update functions, we may simplify the results by assuming that the filter adapts at a slow enough rate for small value of  $N_{ap}$  to justify making the following approximations [3], [18]

$$\frac{\partial J_{MOE}}{\partial a_{p,j}^*(n)} = u^*(n) \cdot \left( -\frac{\partial [u(n-j)]}{\partial a_{p,j}^*(n)} \right) \approx u^*(n) \cdot \left( -\frac{\partial [u(n-j)]}{\partial a_{p,j}^*(n-j)} \right)$$

$$= -u^*(n) \cdot u(n-j), \text{ for } j = 1 \sim N_{ap}$$

According to (12) and (13), the ZPWF can be recursively updated by using

$$a_{z,i}(n+1) = a_{z,i}(n) - \mu_{w1} \cdot u^*(n) \cdot t(n-i), \text{ for } i = 1 \sim N_{az}$$

(14)

$$a_{p,j}(n+1) = a_{p,j}(n) + \mu_{w1} \cdot u^*(n) \cdot u(n-j), \text{ for } j = 1 \sim N_{ap}$$

(15)

where  $a_{z,i}(0) = 0$ , for  $i = 1 \sim N_{az}$  and  $a_{p,j}(0) = 0$ , for  $j = 1 \sim N_{ap}$ .

(c) Blind Equalizer (BE): The constant modulus algorithm (CMA) proposed by Godard [22] will be employed in the BE whose cost function is  $J_G = E\{[|y(n)|^2 - R_2]^2\}$ , where  $R_2 \equiv E\{|s(n)|^4\} / E\{|s(n)|^2\}$ . The output of BE is  $y(n) = \sum_{i=0}^{N_b} b_i^*(n) \cdot u(n-i)$ , where tap weights  $b_i(n)$  may be recursively updated by

$$b_i(n+1) = b_i(n) - \mu_{b1} \cdot \left( |y(n)|^2 - R_2 \right) \cdot y^*(n) \cdot u(n-i),$$

for  $i = 0 \sim N_b$  (16)

where  $\mu_{b1}$  is step-size parameter, and  $[b_0(0), \dots, b_{N_b}(0)]^T = [0, \dots, 0, 1, 0, \dots, 0]^T$ .

### B. Adaptive Blind DFE using ZPWF in Tracking Mode

In order to obtain the structure for the ZPWF in the

tracking mode shown in Fig. 5, which turns out to be equivalent to that of Fig. 2, the following formulation may be computed from (9)

$$A(z) = \frac{1+A_z(z)}{1+A_p(z)} = \frac{1+A_z(z)}{1+A_z(z)+A_p(z)-A_z(z)} = \frac{1}{1+\frac{A_p(z)-A_z(z)}{1+A_z(z)}} \quad (17)$$

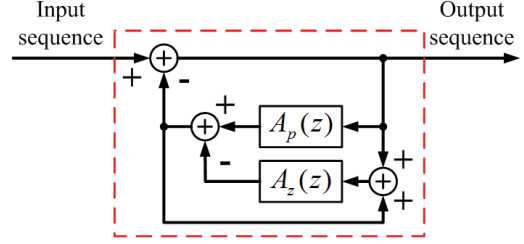


Fig. 5. ZPWF structure equivalent to that of Fig. 2.

When eye is open (in the tracking mode), the position of the ZPWF is interchanged with the position of the original BE in Fig. 4, which is referred to as the decision-directed equalizer (DDE) in the tracking mode as depicted in Fig. 6. The use of the proposed structure of ZPWF in Fig. 5 in the tracking mode along with the decision device being enclosed will be referred to herein as ZPWF(DFE) in Fig. 6. One unique feature of the proposed ZPWF(DFE) can be explained as follows. The presence of a large feedback filter length in the conventional DFE is known to be a major culprit causing error propagation [5]. Therefore, constraining the feedback filter length may improve the DFE performance. The proposed ZPWF(DFE), which approximates a long feedback filter with a smaller number of poles and zeros implemented by  $A_p(z)$  and  $A_z(z)$ , respectively, may not only mitigate error propagation but also reduce the complexity of the computation.

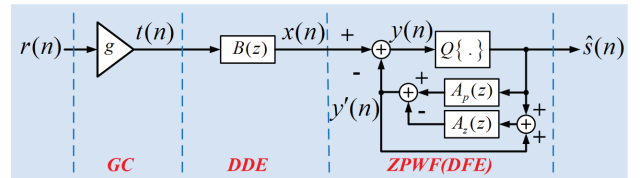


Fig. 6. The adaptive blind DFE using the ZPWF in the tracking mode.

The adaptive blind DFE in the starting mode shown in Fig. 4 may approach a MMSE equalizer once it converges. Accordingly, the phenomenon of error propagation in the DFE may well be prevented once the eye is open. Then the adaptive blind DFE may be switched from the starting mode to the tracking mode to further reduce the MSE especially in severe fading channels to achieve the minimum MSE-DFE solution. Moreover, in communication systems, channels are time varying or change suddenly that may close the eye diagram. Then the adaptive blind DFE would switch back to the starting mode from the tracking mode. Therefore a performance measure, which is an estimate of the true output MSE, is needed to determine when the switching will be necessary between the two modes. We adopt the decision-directed MSE (DDMSE) proposed in [3]-[4] that employs the following recursive formula

$$M_{DD}(n+1) = \lambda M_{DD}(n) + (1-\lambda) |\hat{s}(n) - y(n)|^2 \quad (18)$$

where  $\lambda$  is the forgetting factor and  $M_{DD}(0) = 1$ . In our simulations,  $\lambda = 0.99$  was used. When the adaptive blind DFE is functioning in the starting mode, it switches to the tracking mode provided that  $M_{DD}(n) < M_{th1}$ . However, when the adaptive blind DFE is functioning in the tracking mode, it switches back to starting mode provided that  $M_{DD}(n) > M_{th2}$ . Notably,  $M_{th1} < M_{th2}$  has been chosen to avoid the switching back and forth between the two modes too frequently. Also note that the values of the tap weights of the ZPWF and the BE in the starting mode will be retained in the ZPWF(DFE) and DDE in the tracking mode, and vice versa, respectively, once the switching between the two modes arises. The tracking mode of the adaptive blind DFE is shown in Fig. 6 and the algorithms corresponding to the three stages can be described as follows.

(a) Gain Control (GC): In the tracking mode, the real gain control can simply use a fixed value,  $g$ , computed from the last iteration of the starting mode such that its output may be computed by  $t(n) = g \cdot s(n)$ .

(b) DD Equalizer (DDE) and ZPWF(DFE): The output of the DDE and the ZPWF(DFE) is  $x(n) = \sum_{i=0}^{N_b} b_i^*(n) \cdot t(n-i)$  and  $\hat{s}(n) = Q\{y(n)\}$ , respectively, where  $y(n) = x(n) - y'(n)$  in which  $y'(n) = \left[ \sum_{j=1}^{N_{ap}} a_{p,j}^*(n) \cdot \hat{s}(n-j) - \sum_{i=1}^{N_{az}} a_{z,i}^*(n) \cdot (\hat{s}(n-i) + y'(n-i)) \right]$ . Adaptive implementation of the tap weights of both the DDE and the ZPWF(DFE) using the SGD method can be obtained by jointly minimizing the decision-directed (DD) cost function  $J_{DD} = E\{|\hat{s}(n) - y(n)|^2\}$ . The DDE and the ZPWF(DFE) can then be recursively updated by using

$$\begin{aligned} b_i(n+1) &= b_i(n) - \mu_{b2} \cdot \frac{\partial J_{DD}}{\partial b_i^*(n)} \\ &= b_i(n) + \mu_{b2} \cdot [\hat{s}(n) - y(n)]^* \cdot t(n-i), \quad (19) \\ &\text{for } i = 0 \sim N_b \end{aligned}$$

$$\begin{aligned} a_{z,i}(n+1) &= a_{z,i}(n) - \mu_{w2} \cdot \frac{\partial J_{DD}}{\partial a_{z,i}^*(n)} \\ &= a_{z,i}(n) + \mu_{w2} [\hat{s}(n) - y(n)]^* \cdot (\hat{s}(n-i) + y'(n-i)), \quad (20) \\ &\text{for } i = 1 \sim N_{az} \end{aligned}$$

$$\begin{aligned} a_{p,j}(n+1) &= a_{p,j}(n) - \mu_{w2} \cdot \frac{\partial J_{DD}}{\partial a_{p,j}^*(n)} \\ &= a_{p,j}(n) - \mu_{w2} \cdot [\hat{s}(n) - y(n)]^* \cdot \hat{s}(n-j), \quad (21) \\ &\text{for } j = 1 \sim N_{ap} \end{aligned}$$

where  $\mu_{w2}$  and  $\mu_{b2}$  are step-size parameter.

#### IV. COMPUTER SIMULATIONS

Computer simulations are conducted to compare the adaptive blind DFE using the proposed ZPWF and the existing APWF proposed in [3] and the AZWF proposed in

[4]. The performance measure is based on ensemble-averaged MSE in (3) in dB by averaging over 100 independent trials. The signal constellations considered in the simulations are 16-QAM and signal-and-noise ratio (SNR) is set to be 25dB. Two channels used in the simulations are  $C'(z)$  described in (10) and

$$C''(z) = 0.04 - 0.05z^{-1} + 0.07z^{-2} - 0.21z^{-3} - 0.5z^{-4} + 0.72z^{-5} + 0.36z^{-6} + 0.21z^{-8} + 0.03z^{-9} + 0.07z^{-10}$$

where  $C''(z)$  is a non-minimum phase MA channel from [19]. The z-plane and frequency responses of the two channels are shown in Figs. 7 and 8.

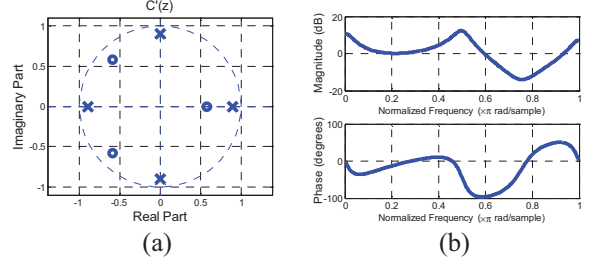


Fig. 7. (a) Z-plane of  $C'(z)$  ("o" and "x" represents zeros and poles, respectively) (b) its magnitude and phase responses.

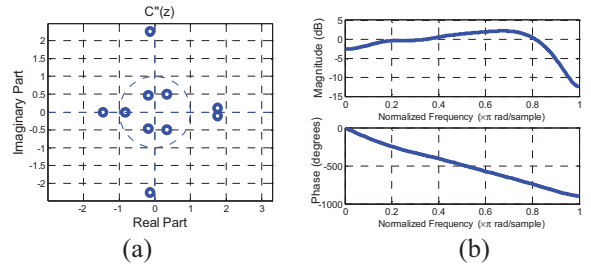


Fig. 8. (a) Z-plane of  $C''(z)$  (b) its magnitude and phase responses.

The two thresholds in our simulations are chosen to be  $M_{th1} = 0.3162$ , (i.e., -5dB), and  $M_{th2} = 0.3981$ , (i.e., -4dB). The equalizer length of BE (DDE) used in all three adaptive blind DFEs are  $N_b + 1 = 21$  and  $N_b + 1 = 11$ , respectively, when channels  $C'(z)$  and  $C''(z)$  are used. In all simulations, the step-sizes of the three adaptive blind DFEs are chosen such that they generated about the same steady-state MSE. Figure 9(a) demonstrates the MSE performances of the three DFEs using the ZPWF ( $N_{az} = 4$ ,  $N_{ap} = 3$ ), AZWF ( $N_{az} = 20$ ) and APWF ( $N_{ap} = 20$ ) with  $C'(z)$ . The adaptive blind DFE employing the ZPWF yielded the fastest convergence speed than those of using the APWF and the AZWF. Furthermore, the ZPWF in the adaptive blind DFE requires smaller number of tap weights than the other two DFEs, which confirms that the ZPWF can be more economically modeled with both poles and zeros than APWF and AZWF when the ARMA channel is used. Figure 9(b) demonstrates the MSE performances of the three adaptive blind DFEs using ZPWF ( $N_{az} = 5$ ,  $N_{ap} = 5$ ), AZWF ( $N_{az} = 10$ ), and APWF ( $N_{ap} = 5$ ) with  $C''(z)$ . The APWF generated

fastest convergence speed among the three DFEs, which confirms the fact that the APWF is most suitable to compensate for the distortion caused by the MA channel  $C''(z)$  (see also Section II). Although the ZPWF uses the same filter length as that of the AZWF, the rate of convergence of the former is faster than that of the latter. Unsurprisingly, the ZPWF is expected to display faster rate of convergence than that of the APWF when an AR channel is used (not shown in this paper).

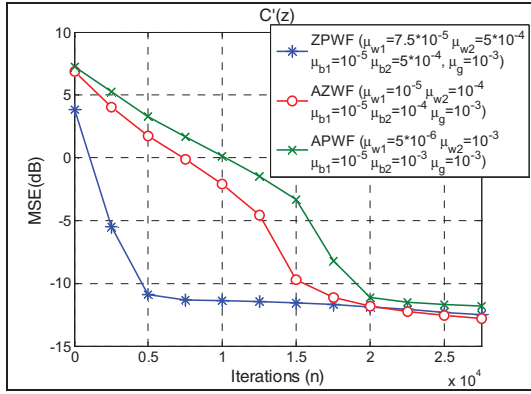


Fig. 9(a). Comparison of MSE performances among the three adaptive blind DFEs when using Channel  $C'(z)$ .

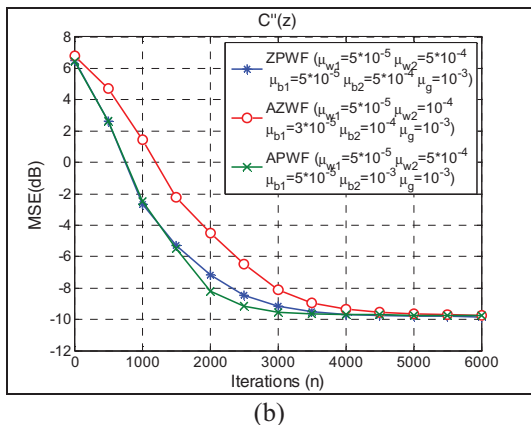


Fig. 9(b). Comparison of MSE performances among the three adaptive blind DFEs when using Channel  $C''(z)$ .

## V. CONCLUSION

This paper proposes an adaptive blind DFE using the ZPWF for amplitude equalization along with a blind equalizer for phase equalization in the starting mode to avoid error propagation as well as using a DD equalizer (DDE) along with the ZPWF (DFE) based on the output of the decision device in the tracking mode to approach the minimum MSE-DFE solution. The adaptive blind DFE using the ZPWF with the least number of tap weights is most suitable to compensate for the amplitude distortion caused by the ARMA channel. For the AR and the MA channels, the proposed adaptive blind DFE using the ZPWF still outperforms the conventional adaptive blind DFE using the APWF and AZWF, respectively. Computer simulations have also been conducted to verify these results.

## REFERENCES

- [1] M. Ghosh, "Blind decision feedback equalization for terrestrial television receivers," *Proc. IEEE*, vol. 86, no. 10, pp. 2070-2081, Oct. 1998.
- [2] M. Ghosh, "Analysis of the MMSE DFE with error propagation," in *Proc. IEEE GLOBECOM Commun. Theory Mini-Conf*, vol. 4, pp. 85-89, Nov. 1997.
- [3] J. Labat, O. Macchi, and C. Laot, "Adaptive decision feedback equalization: Can you skip the training period?" *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 921-930, Jul. 1998.
- [4] W. G. Lim, R. A. Kennedy, and T. D. Abhayapala, "Adaptive decision feedback equalization under parallel adaptation," in *Proc. IEEE Int. Conf. Commun. (ICC '08)*, pp. 495-499, May 2008.
- [5] R. Wang, G. Y. Delisle, "Mitigating error propagation of MMSE-DFE by joint parameter optimization," *IEEE Trans. Commun.*, vol. 57, no. 8, pp. 2239-2243, Aug. 2009.
- [6] R. Lopez-Valcarce, "Realizable linear and decision feedback equalizers: Properties and connections," *IEEE Trans. Signal Process.*, vol. 52, no. 3, pp. 757-773, Mar. 2004.
- [7] C. R. Johnson, Jr., "Adaptive IIR filtering: Current results and open issues," *IEEE Trans. Inf. Theory*, vol. IT-30, no. 2, pp. 237-250, Mar. 1984.
- [8] J. R. Barry, E. A. Lee, and D. G. Messerschmitt, *Digital Communication*, 2nd ed. Boston: Kluwer Academic Publishers, 2004.
- [9] M. Chiani, "Erasures and error propagation in decision-feedback equalization," in *Proc. ISITA*, pp. 1295-1299, Nov. 1994.
- [10] B. Koo and J. D. Gibson, "Experimental comparison of all-pole, all-zero, and pole-zero predictors for ADPCM speech coding," *IEEE Trans. Commun.*, vol. COM-34, pp. 285-290, Mar. 1986.
- [11] M. Bonnet, O. Macchi and M. Jaidane-Saidane, "Theoretical analysis of the ADPCM CCITT algorithm," *IEEE Trans. Commun.*, vol. 38, pp.847-858, June 1990.
- [12] M. G. Larimore, I. R. Treichler, and C. R. Johnson, Jr., "SHARF: An algorithm for adapting IIR digital filters," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-28, pp. 428-440, Aug. 1980.
- [13] I. D. Landau, L. Dugard, and S. Cabrera, "Applications of output error recursive estimation algorithms for adaptive signal processing," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (IEEE ICASSP)*, vol. 7, pp. 639-642, May 1982.
- [14] M. Sternad and A. Ahlen, "The structure and design of realizable decision feedback equalizers for IIR channels with colored noise," *IEEE Trans. Inf. Theory*, vol. 36, no. 4, pp.848-858, Jul. 1990.
- [15] P. M. Crespo and M. L. Honig, "Pole-zero decision feedback equalization with a rapidly converging adaptive IIR algorithm," *IEEE J. Sel. Areas Commun.*, vol. 9, no. 6, pp. 817-829, Aug. 1991.
- [16] N. Al-Dhahir, A. H. Sayed and J. M. Cioffi, "Stable pole-zero modeling for long FIR filters with applications to the MMSE-DFE," *IEEE Trans. Commun.*, vol. 45, no. 5, pp. 508-513, May 1997.
- [17] Z. Ding and Ye Li, "ARMA System Identification Based on Second-Order Cyclostationarity," *IEEE Trans. Signal Process.*, vol. 42, pp. 3483-3494, Dec. 1994.
- [18] R. A. Casas, T. J. Endres, A. Touzni, C. J. Richard Jr., and J. R. Treichler, "Current approaches to blind decision feedback equalization," in *Signal Processing Advances in Wireless & Mobile Communications*, G. B. Giannakis, Y. Hua, P. Stoica, and L. Tong, Eds. Englewood Cliffs, NJ: Prentice-Hall, 2001, ch. 11.
- [19] J. G. Proakis, *Digital Communications*, 3rd. ed. New York: McGrawHill, 1995.
- [20] J. Labat and C. Laot, "Blind Adaptive Multiple-Input Decision-Feedback Equalizer with a Self-Optimized Configuration," *IEEE Trans. Commun.*, vol. 49, no. 4, pp.646-654, Apr. 2001.
- [21] S. Haykin, *Adaptive Filter Theory*, Englewood Cliffs, NJ: Prentice Hall, 2002.
- [22] D. N. Godard, "Self-recovering equalization and carrier tracking in two-dimensional data communication system," *IEEE Trans. Commun.*, vol. COM-28, pp.1867-1875, Nov. 1980.