ANALYSIS OF THE MULTIMODULUS BLIND EQUALIZATION ALGORITHM FOR CROSS QAM SIGNAL CONSTELLATIONS

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Abstract

This work mathematically analyzes the multimodulus blind equalization algorithm (MMA) for cross quadrature amplitude modulation (QAM) signal constellations proposed by Yang, Werner, and Dumont. The analysis demonstrates that the MMA implicitly incorporates a phase-tracking loop, which can remove inter-symbol interference (ISI) while simultaneously correcting the phase error. The analysis also indicates that the decision device may yield wrong estimated symbols unless an appropriate compensation technique is incorporated into the algorithm.

1. Introduction

Adaptive channel equalization without a training sequence is known as blind equalization [1] – [7]. The major advantage of such a technique is that no training sequence is required to start or restart the system when the communication unexpectedly breaks down. Figure 1 illustrates an equivalent baseband model with a channel impulse response of $c(n)$. The channel input, additive white Gaussian noise, and equalizer input are denoted by $s(n)$, $w(n)$, and $u(n)$, respectively. The data symbols transmitted, $s(n)$, is assumed to consist of stationary independently and identically distributed (i.i.d.), real or complex non-Gaussian random variables belonging to a finite alphabet A. The channel is possibly a nonminimum phase linear time-invariant filter. The equalizer input, $u(n)=s(n)*c(n)+w(n)$ is then sent to a tap-delay-line blind equalizer intended to equalize the distortion caused by inter-symbol interference (ISI) without a training signal, where $*$ denotes linear convolution. The output of the blind equalizer $y(n)=s(n)*h(n)+w(n)*f(n)$ can be used to recover the transmitted data symbols, $s(n)$, where $h(n)=c(n)*f(n)$ denotes the impulse response of the combined channel-equalizer system.

The constant modulus algorithm (CMA) [1] for blind equalization is well known to require a separate carrier recovery system (for example, a phase-locked loop) for phase recovery, because the CMA cost function is invariant under a phase rotation in the constellation. Oh and Chin [6], and Yang, Werner, and Dumont [8] proposed a modified CMA called the multimodulus algorithm (MMA) to solve the problem of this arbitrary phase rotation inherent in the CMA. Their modified algorithm has the following cost function

$$J_{MMA}=J_R(n)+J_I(n)=E\{[y_R^2(n)-R_{2,R}]^2\}+E\{[y_I^2(n)-R_{2,I}]^2\} \quad (1)$$

where $y_R(n)$ and $y_I(n)$ are the real and imaginary parts of the equalizer output, respectively; $R_{2,R}$ and $R_{2,I}$ are given by $R_{2,R}=\frac{E\{s_R^4(n)\}}{E\{s_R^2(n)\}}$ and $R_{2,I}=\frac{E\{s_I^4(n)\}}{E\{s_I^2(n)\}}$, in which $s_R(n)$ and $s_I(n)$ denote the real and imaginary parts of $s(n)$, respectively. Tsai and Yuan [12] analyzed the MMA for *square* quadrature amplitude modulation (QAM) signal constellations. Their analysis indicated that the MMA alone may be able to remove inter-symbol interference (ISI) and simultaneously correct the phase error, because it implicitly incorporates a phase-tracking loop, which automatically recovers carrier phase.

This study analyzes the MMA when the transmitted symbol statistics are of QAM *nonsquare* (or *cross*) constellations for which the number of bits per symbol is odd [pp. 372, 9], [pp.280, 10]. As an example, Fig. 2 depicts the cross constellation for a 32-QAM input. This constellation is obtained from a square constellation of 6×6=36 points, by removing one outer point in each corner. Clearly, the resulting statistics of the source differ from those derived in [12] for square signal constellations. For example, the real and imaginary parts of the source statistics are no longer independent (i.e. $E\{s_R(n)s_I(n)\}\neq E\{s_R(n)\}E\{s_I(n)\}$).

2. Analysis of MMA for QAM Cross Constellations

A. Derivation of the MMA for QAM Cross Constellations

Yang, Werner, and Dumont [8] modified the MMA in (1) to take advantage of the statistics of the symbols used in *cross constellations*. The in-phase (or real) and quadrature (or imaginary) cost functions for square constellations in (1) are modified as follows for cross constellations having two different sets of statistics along each dimension:

$$
\begin{aligned}
J_{MMA,R_1}(n) &= E\left\{\left(y_R^2(n) - R_1\right)^2\right\}, \text{ if } |y_I(n)| < \delta \\
J_{MMA,R_2}(n) &= E\left\{\left(y_R^2(n) - R_2\right)^2\right\}, \text{ if } |y_I(n)| > \delta \\
J_{MMA,I_1}(n) &= E\left\{\left(y_I^2(n) - R_1\right)^2\right\}, \text{ if } |y_R(n)| < \delta \\
J_{MMA,I_2}(n) &= E\left\{\left(y_I^2(n) - R_2\right)^2\right\}, \text{ if } |y_R(n)| > \delta
\end{aligned} \qquad (2)
$$

where δ is a constant that is a function of the signal constellation under consideration. Notice that two different constants, R_1 and R_2, are used in (2). As stated in [8], a single modulus can also be used, but doing so would increase the probability of converging to the so-called "wrong solution". However, the analysis herein indicates that the MMA given by (2) for cross constellations may also yield wrong estimated symbols, $\hat{s}(n)$, unless a suitable compensation technique is provided, owing to the use of two different sets of statistics along each dimension. As will be shown later, an automatic gain control (AGC) can be employed to compensate appropriately for the equalizer output, $y(n)$, such that the decision device can generate the correct estimated symbol, $\hat{s}(n)$.

Using the unit step function $U(n)=\begin{cases}0 & ,n<0\\ 1 & ,n\geq 0\end{cases}$, the MMA cost function for cross constellations [which is a weighted sum of the four cost functions shown in (2)], can be expressed as,

$$
\begin{aligned}
J_{MMA}(n) &= E\left\{\left(y_R^2(n) - R_1\right)^2\right\}\left[1-U(|y_I(n)|-\delta)\right] \\
&\quad + E\left\{\left(y_I^2(n) - R_1\right)^2\right\}\left[1-U(|y_R(n)|-\delta)\right] \\
&\quad + E\left\{\left(y_R^2(n) - R_2\right)^2\right\}\left[U(|y_I(n)|-\delta)\right] \\
&\quad + E\left\{\left(y_I^2(n) - R_2\right)^2\right\}\left[U(|y_R(n)|-\delta)\right] \\
&= E\left\{\left(y_R^2(n)-R_1\right)^2\right\}+E\left\{\left(y_I^2(n)-R_1\right)^2\right\} \\
&\quad +U(|y_I(n)|-\delta)\left[E\left\{\left(y_R^2(n)-R_2\right)^2\right\}-E\left\{\left(y_R^2(n)-R_1\right)^2\right\}\right] \\
&\quad +U(|y_R(n)|-\delta)\left[E\left\{\left(y_R^2(n)-R_2\right)^2\right\}-E\left\{\left(y_I^2(n)-R_1\right)^2\right\}\right]
\end{aligned} \qquad (3)
$$

As in [12], the MMA for QAM cross constellations is analyzed for a complex i.i.d., zero-mean source and a complex baseband channel, by excluding the additive channel noise. The assumptions of each member of the symbol alphabet being equiprobable in the source sequence and of equalizer being either doubly-infinite in length or of finite-length fractionally spaced remain valid. A substantial amount of algebraic manipulation yields the general formulation of the MMA cost function for cross constellations in (3) as

$$
\begin{aligned}
&J_{MMA}(n)= \\
&\tfrac{1}{4}\mathrm{Re}\left\{E\{s^4(n)\}\sum_i h^4(i)+3E\{s^2(n)\}^2\sum_i\sum_{l\neq i}h^2(i)h^2(l)\right\} \\
&+\tfrac{3}{4}\left(k_s\sigma_s^4\sum_i|h(i)|^4+2\sigma_s^4\sum_i\sum_{l\neq i}|h(i)|^2|h(l)|^2+|E\{s^2(n)\}|^2\sum_i\sum_{l\neq i}h^2(i)\left(h^2(l)\right)^*\right) \\
&-2R_1\cdot\sigma_s^2\left(\sum_i|h(i)|^2\right)+2R_1^2 \\
&+U(|y_I(n)|-\delta)\left(2(R_1-R_2)\left[\tfrac{1}{2}\mathrm{Re}\left\{E\{s^2(n)\}\sum_i h^2(i)\right\}+\tfrac{1}{2}\cdot\sigma_s^2\sum_i|h(i)|^2\right]+\left(R_2^2-R_1^2\right)\right) \\
&+U(|y_R(n)|-\delta)\left(2(R_1-R_2)\left[-\tfrac{1}{2}\mathrm{Re}\left\{E\{s^2(n)\}\sum_i h^2(i)\right\}+\tfrac{1}{2}\cdot\sigma_s^2\sum_i|h(i)|^2\right]+\left(R_2^2-R_1^2\right)\right),
\end{aligned} \qquad (4)
$$

where $\sigma_s^2=E\{|s(n)|^2\}$, $k_s=\frac{E\{|s(n)|^4\}}{\sigma_s^4}$ and $h(k)=h_R(k)+jh_I(k)=r(k)e^{j\theta(k)}$ is the kth position of the combined channel-equalizer impulse response vector **h**=[…h(-1),h(0),h(1)…] and $r(k)=\sqrt{h_R^2(k)+h_I^2(k)}, \theta(k)=\tan^{-1}\frac{h_I(k)}{h_R(k)}$. Equation (4) can be further simplified by realizing that $E\{s^2(n)\}=0$ and $E\{s^4(n)\}$ is real, so (4) is reduced to

$$
\begin{aligned}
J_{MMA}(n) &= \Big[\tfrac{1}{4}\mathrm{Re}\left\{E\{s^4(n)\}\sum_i h^4(i)\right\} \\
&\quad +\tfrac{3}{4}\left(k_s\sigma_s^4\sum_i|h(i)|^4+2\sigma_s^4\sum_i\sum_{l\neq i}|h(i)|^2|h(l)|^2\right) \\
&\quad -2R_1\cdot\sigma_s^2\left(\sum_i|h(i)|^2\right)+2R_1^2\Big] \\
&\quad +\left((R_1-R_2)\left(\sigma_s^2\sum_i|h(i)|^2\right)+\left(R_2^2-R_1^2\right)\right)\left(U(|y_I(n)|-\delta)+U(|y_R(n)|-\delta)\right)
\end{aligned} \qquad (5)
$$

Notably, the first term, $\tfrac{1}{4}\mathrm{Re}\left[E\{s^4(n)\}\sum_i h^4(i)\right]$, which contains the phase information of the blind equalizer output, still exists in the cross constellation case. As mentioned in [12], this phase information enables a possible phase error to be removed. Accordingly, the MMA for QAM cross constellations, like its square counterpart, may remove phase jitter without the use of a separate carrier tracking loop. This situation is in contrast to that of the CMA, whose cost function is insensitive to the phase of the equalizer output, such that the CMA alone cannot achieve phase recovery, causing an additional carrier recovery system (such as a phase-locked loop) to be required for phase recovery.

B. Stationary Points of the MMA for Cross Constellations

The term, $U(|y_I(n)|-\delta)+U(|y_R(n)|-\delta)$, takes three possible values, which are 0, 1, and 2. These three values result in the following three MMA cost functions at each iteration, n, denoted respectively, by $J_{MMA0}(n)$, $J_{MMA1}(n)$, and $J_{MMA2}(n)$ according to the values of the real and imaginary parts of the equalizer output, $y(n)$:

$$J_{MMA,0}(n)=\frac{1}{4}\mathrm{Re}\left\{E\{s^4(n)\}\sum_i h^4(i)\right\}+\frac{3}{4}\left(k_s\sigma_s^4\sum_i|h(i)|^4+2\sigma_s^4\sum_i\sum_{l\neq i}|h(i)|^2|h(l)|^2\right)$$
$$-2R_1\cdot\sigma_s^2\left(\sum_i|h(i)|^2\right)+2R_1^2$$
$$J_{MMA,1}(n)=\frac{1}{4}\mathrm{Re}\left\{E\{s^4(n)\}\sum_i h^4(i)\right\}+\frac{3}{4}\left(k_s\sigma_s^4\sum_i|h(i)|^4+2\sigma_s^4\sum_i\sum_{l\neq i}|h(i)|^2|h(l)|^2\right)$$
$$-(R_1+R_2)\sigma_s^2\left(\sum_i|h(i)|^2\right)+\left(R_1^2+R_2^2\right)$$
$$J_{MMA,2}(n)=\frac{1}{4}\mathrm{Re}\left\{E\{s^4(n)\}\sum_i h^4(i)\right\}+\frac{3}{4}\left(k_s\sigma_s^4\sum_i|h(i)|^4+2\sigma_s^4\sum_i\sum_{l\neq i}|h(i)|^2|h(l)|^2\right)$$
$$-2R_2\cdot\sigma_s^2\left(\sum_i|h(i)|^2\right)+2\cdot R_2^2 \qquad (6)$$

Clearly, $J_{MMA,0}(n)$, $J_{MMA,1}(n)$, and $J_{MMA,2}(n)$ differ from the corresponding terms in the MMA cost function for square constellations ($J_{MMA}(n)$, in (3) of [12]) only in the coefficients of the last two terms. Consequently, a discrete-time first-order phase-locked loop, hidden inside the MMA for square signal constellations, remains in the MMA for cross signal constellations. The general form of all possible stationary points of the MMA for cross constellations is now derived. Without any loss of generality, only $J_{MMA,1}(n)$ in (6) is considered, such that $\nabla J_{MMA,1}(n)=0$, yielding

$$E\{s^4(n)\}r^3(k)\cos 4\theta(k)+3k_s\sigma_s^4r^3(k)+6\sigma_s^4r(k)\sum_{i\neq k}r^2(i) \qquad (7)$$
$$-2(R_1+R_2)\sigma_s^2r(k)=0$$

$$E\{s^4(n)\}\cdot r^3(k)[-\sin 4\cdot\theta(k)]=0 \qquad (8)$$

Equation (8) is crucial as it restricts $\theta(k)$ to only eight different values. Equation (7) yields one result when $\theta(k)\in\left\{0,\frac{\pi}{2},\pi,\frac{3\pi}{2}\right\}$ (such that $\cos 4\theta(k)=1$) and another when $\theta(k)\in\left\{\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\}$ (such that $\cos 4\theta(k)=-1$). Therefore, all of the possible equilibria of the MMA can be grouped into two categories (i.e., $\theta(k)\in\left\{0,\frac{\pi}{2},\pi,\frac{3\pi}{2}\right\}$ and $\theta(k)\in\left\{\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\}$) according to the values of $\cos 4\theta(k)$. A general form for all possible stationary points of the MMA is now derived for both transient and steady-state mode operations. Substituting $\theta(k)\in\left\{0,\frac{\pi}{2},\pi,\frac{3\pi}{2}\right\}$ (or $\cos 4\theta(k)=1$) into (7) yields

$$\left[E\{s^4(n)\}+3k_s\sigma_s^4\right]r_+^3(k)+\left[6\sigma_s^4\sum_{i\neq k}r_+^2(i)-2(R_1+R_2)\sigma_s^2\right]r_+(k)=0,\ k=1,\dots,M \qquad (9)$$

Substituting $\theta(k)\in\left\{\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\}$ (or $\cos 4\theta(k)=-1$) into (7) yields

$$\left[-E\{s^4(n)\}+3k_s\sigma_s^4\right]r_-^3(k)+\left[6\sigma_s^4\sum_{i\neq k}r_-^2(i)-2(R_1+R_2)\sigma_s^2\right]r_-(k)=0,\ k=1,\dots,M \qquad (10)$$

Clearly, (9) and (10) give $r_+^2(1)=r_+^2(2)=\dots=r_+^2(M)$ and $r_-^2(1)=r_-^2(2)=\dots=r_-^2(M)$, respectively. Consequently, (9) and (10) given $k=M$ suffice to determine both $r_+^2(M)$ and $r_-^2(M)$:

$$\left[E\{s^4(n)\}+3k_s\sigma_s^4\right]r_+^3(M)+\left[6\sigma_s^4\sum_{i\neq k}r_+^2(i)-2(R_1+R_2)\sigma_s^2\right]r_+(M)=0 \qquad (11)$$

$$\left[-E\{s^4(n)\}+3k_s\sigma_s^4\right]r_-^3(M)+\left[6\sigma_s^4\sum_{i\neq k}r_-^2(i)-2(R_1+R_2)\sigma_s^2\right]r_-(M)=0 \qquad (12)$$

Using $E[s^4(n)]=2E[s_R^4(n)]-6E^2[s_R^2(n)]$, $k_s\sigma_s^4=2E[s_R^4(n)]+2E^2[s_R^2(n)]$, and $\sigma_s^4=4E^2[s_R^2(n)]$, yields $r_+^2(M)=\frac{(R_1+R_2)\sigma_s^2}{4E\{s_R^4(n)\}+[12(M-1)]E^2\{s_R^2(n)\}}$ and $r_-^2(M)=\frac{(R_1+R_2)\sigma_s^2}{2E\{s_R^4(n)\}+[6(2M-1)]E^2\{s_R^2(n)\}}$, which are directly solved by using (11) and (12), respectively. Other cases ($J_{MMA,0}(n)$ and $J_{MMA,2}(n)$) can be derived similarly. The following general form for all possible stationary points of $J_{MMA,0}(n)$, $J_{MMA,1}(n)$, and $J_{MMA,2}(n)$ can be obtained

$$|h_M(k)|^2=\left[r_+^2(M)\sum_{i\in I_M \text{ and } \theta(k)\in\left\{0,\frac{\pi}{2},\pi,\frac{3\pi}{2}\right\}}\delta(k-i)\right]+\left[r_-^2(M)\sum_{i\in I_M \text{ and } \theta(k)\in\left\{\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\}}\delta(k-i)\right] \qquad (13)$$

where $r_+^2(M)=\frac{(R_1+R_2)\sigma_s^2}{A}$ and $r_-^2(M)=\frac{(R_1+R_2)\sigma_s^2}{B}$; $r_+^2(M)=\frac{2R_1\sigma_s^2}{A}$ and $r_-^2(M)=\frac{2R_1\sigma_s^2}{B}$, and $r_+^2(M)=\frac{2R_2\sigma_s^2}{A}$ and $r_-^2(M)=\frac{2R_2\sigma_s^2}{B}$ for $J_{MMA1}(n)$, $J_{MMA0}(n)$, and $J_{MMA2}(n)$, respectively, where $A=4E\{s_R^4(n)\}+12(M-1)E^2\{s_R^2(n)\}$ and $B=2E\{s_R^4(n)\}+6(2M-1)E^2\{s_R^2(n)\}$.

The Foschini method [7] can also be used to confirm that, if the equalizer is doubly infinite and the distribution of $s(n)$ is sub-Gaussian, then all the prespecified $\mathbf{h}_M$ (with the associated l_M); for $M \geq 2$, are unstable equilibria (saddle points). Herein, only the special case of $M = 1$ is considered (in which case the MMA converges to the ideal form $\mathbf{h}_1 = [0,0,\ldots,0,h_1(k),0,\ldots,0,0]^T$). Hence, (6) reduces to

$$\begin{cases} J_{MMA,0}(n) = \frac{1}{4}E\{s^4(n)\}\sum_i r^4(i)\cos 4\theta(i) + \frac{3}{4}k_s\sigma_s^4\sum_i r^4(i) - 2R_1\sigma_s^2\sum_i r^2(i) + 2R_1^2 \\ J_{MMA,1}(n) = \frac{1}{4}E\{s^4(n)\}\sum_i r^4(i)\cos 4\theta(i) + \frac{3}{4}k_s\sigma_s^4\sum_i r^4(i) - (R_1+R_2)\sigma_s^2\sum_i r^2(i) + (R_1^2+R_2^2) \\ J_{MMA,2}(n) = \frac{1}{4}E\{s^4(n)\}\sum_i r^4(i)\cos 4\theta(i) + \frac{3}{4}k_s\sigma_s^4\sum_i r^4(i) - 2R_2\sigma_s^2\sum_i r^2(i) + 2R_2^2 \end{cases} \quad (14)$$

Table I summarizes the set of stationary points for $J_{MMA,1}(n)$, $J_{MMA,0}(n)$, $J_{MMA,2}(n)$ with $M = 1$.

C. Saddle Points and Desired Global Minima

The second derivative test is applied to determine whether these stationary points are local minima, local maxima, or saddle points. Only the stationary points for $J_{MMA,1}(n)$ are analyzed here. The analyses of the stationary points for both $J_{MMA,0}(n)$ and $J_{MMA,2}(n)$ are similar, and the results are also summarized in Table I. The following derivatives can be computed first by considering $J_{MMA,1}(n)$ in (14):

$$\begin{cases} \frac{\partial^2 J_{MMA,1}}{\partial h_R^2(k)} = \frac{1}{4}E\{s^4(n)\}\{12h_R^2(k) - 12h_I^2(k)\} + \frac{3}{4}k_s\sigma_s^4\{12h_R^2(k) + 4h_I^2(k)\} - 2(R_1+R_2)\sigma_s^2 \\ \frac{\partial^2 J_{MMA,1}}{\partial h_I^2(k)} = \frac{1}{4}E\{s^4(n)\}\{12h_I^2(k) - 12h_R^2(k)\} + \frac{3}{4}k_s\sigma_s^4\{12h_I^2(k) + 4h_R^2(k)\} - 2(R_1+R_2)\sigma_s^2 \\ \frac{\partial^2 J_{MMA,1}}{\partial h_R(k)\partial h_I(k)} = \frac{1}{4}E\{s^4(n)\}\{-24h_R(k)\cdot h_I(k)\} + \frac{3}{4}k_s\sigma_s^4\{8h_R(k)h_I(k)\} \end{cases} \quad (15)$$

For a sub-Gaussian input, the following can be easily verified using the second derivative test on pp. 768 of [11];

$$\begin{cases} D(0,0) = 4(R_1+R_2)^2\sigma_s^4 > 0 \\ \left[\frac{\partial^2 J_{MMA,1}(h_R(k),h_R(k))}{\partial h_R^2(k)}\right]_{[h_R(k),h_I(k)]=[0,0]} = -2(R_1+R_2)\sigma_s^2 < 0 \end{cases} \quad (16)$$

so $[h_R(k), h_I(k)] = [0,0]$ is a local maximum. Similarly, for $r^2(k) = \frac{2(R_1+R_2)\sigma_s^2}{E\{s(n)^4\}+3k_s\sigma_s^4}$ with $\theta(k) \in \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$, the following can be verified;

$$\begin{cases} D[h_R(k),h_I(k)] = 4(R_1+R_2)\sigma_s^2\left\{-4E\{s^4(n)\}\frac{2(R_1+R_2)\sigma_s^2}{E\{s(n)^4\}+3k_s\sigma_s^4}\right\} > 0 \\ \left[\frac{\partial^2 J_{MMA,1}(h_R(k),h_R(k))}{\partial h_R^2(k)}\right]_{r^2(k)=\frac{2(R_1+R_2)\sigma_s^2}{E\{s(n)^4\}+3k_s\sigma_s^4}} = -4E\{s^4(n)\}\frac{2(R_1+R_2)\sigma_s^2}{E\{s(n)^4\}+3k_s\sigma_s^4} > 0 \end{cases} \quad (17)$$

so the four stationary points $[h_{1,R}(k), h_{1,I}(k)] = [\pm\sqrt{\frac{2(R_1+R_2)\sigma_s^2}{E\{s(n)^4\}+3k_s\sigma_s^4}}, 0]$ and $[0, \pm\sqrt{\frac{2(R_1+R_2)\sigma_s^2}{E\{s(n)^4\}+3k_s\sigma_s^4}}]$ are local minima. For $r^2(k) = \frac{2(R_1+R_2)\sigma_s^2}{-E\{s(n)^4\}+3k_s\sigma_s^4}$ with $\theta(k) \in \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$, the following can be verified;

$$\begin{cases} D[h_R(k),h_I(k)] = \frac{\left(2(R_1+R_2)\sigma_s^2\right)^2}{-E\{s(n)^4\}+3k_s\sigma_s^4}\left(8E\{s(n)^4\}\right) < 0 \\ \left[\frac{\partial^2 J_{MMA,1}(h_R(k),h_R(k))}{\partial h_R^2(k)}\right]_{r^2(k)=\frac{2(R_1+R_2)\sigma_s^2}{-E\{s(n)^4\}+3k_s\sigma_s^4}} = 2(R_1+R_2)\sigma_s^2\frac{E\{s(n)^4\}+3k_s\sigma_s^4}{-E\{s(n)^4\}+3k_s\sigma_s^4} > 0 \end{cases} \quad (18)$$

which four stationary points are four saddle points. Accordingly,

$$[h_{1,R}(k), h_{1,I}(k)] = [\pm\sqrt{\frac{2(R_1+R_2)\sigma_s^2}{E\{s(n)^4\}+3k_s\sigma_s^4}}, 0] \text{ or } [0, \pm\sqrt{\frac{2(R_1+R_2)\sigma_s^2}{E\{s(n)^4\}+3k_s\sigma_s^4}}]$$

are the only four local (hence global) minima, as indicated in Figs. 3-5 which depict the cost function of $J_{MMA,1}(n)$, $J_{MMA,0}(n)$ and $J_{MMA,2}(n)$ for a 32-QAM input, and its contours in terms of $h_{1,R}(k)$ and $h_{1,I}(k)$, obtained by evaluating $J_{MMA,1}(n)$ in (14).

TABLE I Stationary Points of Cross Constellations (for $M = 1$)

	Local Minima M=1	Saddle Points M=1
$J_{MMA,0}$	$r(k) = \sqrt{\frac{4R_1\sigma_s^2}{E\{s(n)^4\}+3k_s\sigma_s^4}}$ $\theta(k) \in [0, \pi/2, \pi, 3\pi/2]$	$r(k) = \sqrt{\frac{4R_1\sigma_s^2}{-E\{s(n)^4\}+3k_s\sigma_s^4}}$ $\theta(k) \in [\pi/4, 3\pi/4, 5\pi/4, 7\pi/4]$
$J_{MMA,1}$	$r(k) = \sqrt{\frac{2(R_1+R_2)\sigma_s^2}{E\{s(n)^4\}+3k_s\sigma_s^4}}$ $\theta(k) \in [0, \pi/2, \pi, 3\pi/2]$	$r(k) = \sqrt{\frac{2(R_1+R_2)\sigma_s^2}{-E\{s(n)^4\}+3k_s\sigma_s^4}}$ $\theta(k) \in [\pi/4, 3\pi/4, 5\pi/4, 7\pi/4]$
$J_{MMA,2}$	$r(k) = \sqrt{\frac{4R_2\sigma_s^2}{E\{s(n)^4\}+3k_s\sigma_s^4}}$ $\theta(k) \in [0, \pi/2, \pi, 3\pi/2]$	$r(k) = \sqrt{\frac{4R_2\sigma_s^2}{-E\{s(n)^4\}+3k_s\sigma_s^4}}$ $\theta(k) \in [\pi/4, 3\pi/4, 5\pi/4, 7\pi/4]$

D. An AGC for Compensation

In the cross constellation case, for example, for a 32-QAM input source, the dispersion constants R_1 and R_2 are computed from the $6\times6=36$ outer square constellation points and the $4\times4=16$ inner square constellation points depicted in Fig. 2. R_1 and R_2 are computed using different statistics of the 32-QAM constellation while the denominator of $r(k)$ ($E\{s(n)^4\}+3k_s\sigma_s^4$) is computed using the overall statistics of the 32-QAM constellation so the numerator and the denominator of

$r(k)$ of all the local minima of $J_{MMA,0}(n)$, $J_{MMA,1}(n)$, and $J_{MMA,2}(n)$ presented in Table I cannot be cancelled out, as they can in the square constellation case [12]. Consequently, the magnitudes of none of the local minima, $r(k)$, presented in Table I, are unity. This situation is in contrast to that case of the square signal constellation discussed in [12], in which $R_R=R_I=R$ (such that only a single set of statistics applies along each dimension) is computed using the overall statistics of the QAM constellation, such that $r^2(k)=|h_1(k)|^2=(h_{1R}(k))^2+(h_{1I}(k))^2=1$. For a 32-QAM input, the magnitudes of the local minima for $J_{MMA0}(n)$, $J_{MMA1}(n)$, and $J_{MMA2}(n)$ can be computed as $r(k)=\sqrt{\frac{4R_1\sigma_s^2}{E\{|s(n)|^4\}-3k_s\sigma_s^4}}=1.0393$, $r(k)=\sqrt{\frac{2(R_1+R_2)\sigma_s^2}{E\{|s(n)|^4\}+3k_s\sigma_s^4}}=0.8714$, and $r(k)=\sqrt{\frac{4R_2\sigma_s^2}{E\{|s(n)|^4\}+3k_s\sigma_s^4}}=0.6622$, respectively [see also Figs. 3–5]. These results reveal that the final constellations of the equalizer output will be modified by the three cost functions $J_{MMA0}(n)$, $J_{MMA1}(n)$, and $J_{MMA2}(n)$ implying that the magnitude of the equalizer output has been distorted. Consequently, the decision device depicted in Fig. 1 may yield a wrong estimated symbol $\hat{s}(n)$ unless an appropriate compensation technique is incorporated in (2). An AGC for each of the three cost functions is suggested to be employed to compensate for the equalizer output before the decision device makes any decision.

3. Computer Simulations

Computer simulation results are provided to compare the performance of the MMA proposed in [8] for cross QAM constellations between the incorporation of an AGC in (2) and without it. The AGC is employed to adjust the magnitude of the equalizer output such that the magnitude of the combined channel-equalizer impulse response equals unity at each iteration. As illustrated in Fig. 1, the transmitted data symbols $s(n)$ is an independent, identically distributed 32-QAM sequence, and the input to the equalizer $u(n)$ is the sum of the channel output and an independent white Gaussian noise $w(n)$. The real and imaginary parts of the complex-valued additive white Gaussian noise $w(n)$ are assumed to be independent and have equal variance such that the signal noise ratio (SNR) is 30 dB. Simulation experiments described herein employ a complex equalizer of transversal filter structure having 31 tap weights with 15 units of time delay. All the tap weights were initialized by setting the central tap weight to 1 and the others to zero. The step-size parameter, $\mu=10^{-6}$, was used. Plots were generated by ensemble averaging the squared error $|s(n)-y(n)|^2$ (MSE) versus the number of iterations n over 200 independent learning curves. The channel used in the simulations that closely follows from [4] whose impulse response is given by

$$c(t,0.11)W(t)+0.8c(t-0.25T,0.11)W(t-0.25T)-0.4c(t-2T,0.11)W(t-2T)$$

is a raised cosine pulse in a three-ray multipath environment, where $W(t)$ is a rectangular truncation windows spanning $[-3T, 3T]$. Figure 6 shows that the MMA proposed by Yang, Werner, and Dumont in [8] for 32-QAM cross constellations may diverge if an AGC is not incorporated. In contrast, an AGC can compensate for the equalizer output in the steady-state.

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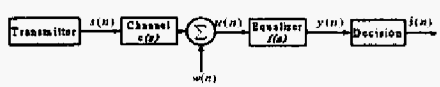


Fig. 1 The simplified baseband model

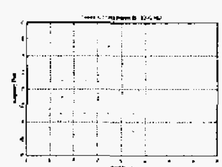
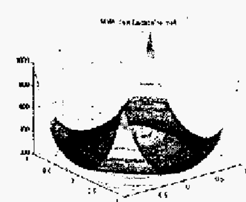


Fig. 2 Cross constellation for a 32-QAM input



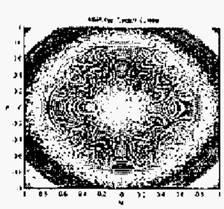


Fig. 3 The cost function of $J_{MMA,1}(n)$ (a) and its contours (b) (for $M=1$)

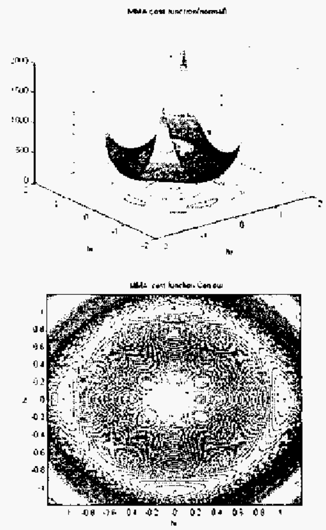


Fig. 4 The cost function of $J_{MMA2}(n)$ (a) and its contours(b) (for $M=1$)

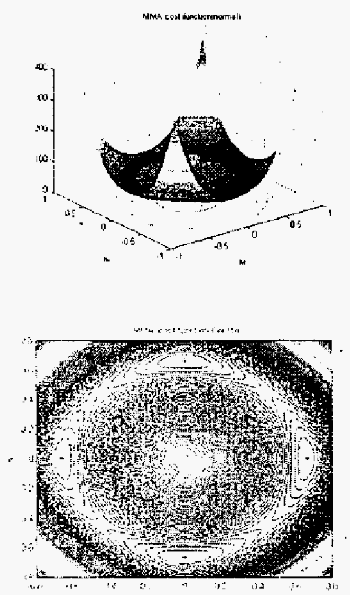


Fig. 5 The cost function of $J_{MMA2}(n)$ (a) and its contours (b) (for $M=1$)

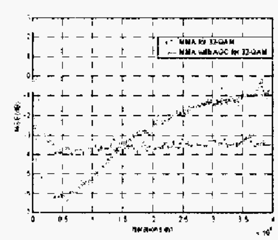


Fig. 6 Learning curves for 32-QAM cross constellation (with and without AGC)

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