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Interference Rejection in DS Spread Spectrum Systems Using Nonlinear Interpolation Lattice Filters

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Abstract—**An M^{th}-order adaptive lattice filter automatically generates all M of the outputs that would be provided by M separate transversal filters. This feature can effectively suppress the narrow-band interference (NBI) of time-varying bandwidth in direct-sequence (DS) spread-spectrum systems for which the order of the interference rejection filter that achieves the optimal performance is unknown and is constantly changing. This work develops a computationally efficient and numerically stable adaptive nonlinear QR-decomposition-based lattice interpolator to effectively suppress NBI. Simulation results indicate that the proposed nonlinear lattice interpolators provide a fast convergence rate and SNR improvements that closely approach the upper bound.**

I. Introduction

Narrow-band interference (NBI) suppression utilizes the discrepancy in the predictability between the interference and the spread-spectrum (SS) signal in that the former can be accurately predicted (or interpolated), whereas the latter is wideband and hence unpredictable. Consequently, a prediction or interpolation of the received signal can be used to estimate the interference. The estimate of the interference can be subtracted from the delayed received signal, thereby suppressing the interference. Previous studies [1], [4] have demonstrated the ability of this preprocessing process to enhance the performance of SS systems. However, when estimating interference, the noise is the sum of a white Gaussian noise and the SS signal that is highly non-Gaussian. This severely degrades the performance of the least mean square (LMS)-based interference suppression. Hence, directly applying the LMS algorithm to suppress the interference cannot produce optimal results. Vijayan and Poor [2] proposed a *nonlinear* LMS-based approximate conditional mean (ACM) [3] predictor capable of performing much better than its linear counterpart. Milstein [1] and Masry [4] demonstrated that better interference rejection may be achieved by using the interpolator rather than the predictor with the same number of taps. This achievement is due to that the former more effectively uses the correlation between the nearest neighboring samples than the latter. The feasibility of using predictor and interpolator employing the *lattice structure* to suppress NBI has been studied in [5], [6] in which simulation and experimental results indicate that lattice structure is highly promising for suppressing NBI, especially in complex jamming environments. The reference [6] employs *linear* QR-decomposition-based least squares lattice (QRD-LSL) interpolator developed in [7] to suppress NBI. This paper develops an adaptive *nonlinear* QRD-LSL interpolator whose computational complexity is proportional to $O(M)$ by incorporating the soft-decision feedback proposed in [2], where M is the order of the interpolator. Simulation results indicate that the proposed nonlinear QRD-LSL interpolator consistently provides excellent results in terms of rate of convergence as well as SNR improvement.

II. Nonlinear ACM Filters for NBI in DS-SS Systems

A. System Model

The spread-spectrum and NBI model used herein is the same as that used in [2], i.e., the received signal (or observation) at sample k is given by

$$z(k) = s(k) + n(k) + i(k) \tag{1}$$

where the ambient white noise $n(k)$ can be modeled as being white Gaussian with variance σ_n^2, the interference $i(k)$ as having a bandwidth much less than the spread bandwidth. In addition, the SS signal $\{s(k)\}$ can be considered to be a sequence of independent and identically distributed (i.i.d.) binary random variables taking on values +1 or −1 with equal probability. Assume that $\{s(k)\}$, $\{w(k)\}$, and $\{i(k)\}$ are mutually independent.

B. Nonlinear ACM Filter

Vijayan and Poor [2] modeled the NBI as a Gaussian autoregressive (AR) process of order q, i.e.,

$$i(k) = \sum_{j=1}^{q} \phi_j i(k-j) + u(k) \tag{2}$$

where $\phi_1, \phi_2, \cdots, \phi_q$ are AR parameters and $\{u(k)\}$ is a white Gaussian process of zero mean and variance σ_u^2. According to (1) and (2), a state-space representation for the received signal and the interference can be expressed by

$$\mathbf{x}(k) = \mathbf{\Phi}\mathbf{x}(k-1) + \mathbf{w}(k) \tag{3}$$

$$\mathbf{z}(k) = \mathbf{H}\mathbf{x}(k) + v(k) \tag{4}$$

where $\mathbf{x}(k) = [i(k)\ i(k-1)\ \cdots\ i(k-q+1)]^T$, $\mathbf{w}(k) = [u(k)\ 0\ \cdots\ 0]^T$, $\mathbf{H} = [1\ 0\ \cdots\ 0]$, $v(k) = s(k) + n(k)$, and $\mathbf{\Phi}$ is the state transition matrix of NBI. Since the first component of the state $\mathbf{x}(k)$ is the interference $i(k)$, an estimate of the state $\mathbf{x}(k)$ gives an estimate of $i(k)$. As well known, the optimal estimate of the state of a system modeled by (3) and (4), in the mean-square error (MSE) sense, is the conditional mean. The predicted estimate of $\mathbf{x}(k)$, $\overline{\mathbf{x}}(k) = E\{\mathbf{x}(k) \mid Z^{k-1}\}$, and the filtered estimate of $\mathbf{x}(k)$, $\widehat{\mathbf{x}}(k) = E\{\mathbf{x}(k) \mid Z^k\}$, at time k, are based on the observations up to time $k-1$ and time k, respectively, where $Z^k = \{z(1), z(2), \cdots, z(k)\}$.

If the received signal $z(k) = i(k) + v(k)$ is used directly as the input to the LMS predictors, $v(k)$, which is highly non-Gaussian, severely degrades the performance of LMS-based interference rejection of $i(k)$. Therefore, an optimal filtered estimate of the interference $i(k)$ based on Z^k, $\widehat{i}(k)$, i.e., referred to herein as *interference estimate*, must be obtained and then used as the input to the LMS predictors instead. Masreliez [3] developed an ACM filter for estimating the state of a linear system with a non-Gaussian measurement noise by making the approximating assumption that the state prediction density $p[\mathbf{x}(k) \mid Z^{k-1}]$ is Gaussian. Therefore, by assuming that the prediction density $p[i(k) \mid Z^{k-1}]$ is Gaussian, given the observations Z^k, with non-Gaussian measurement noise $v(k)$, the interference estimate $\widehat{i}(k) = E\{i(k) \mid Z^k\}$ can be obtained by employing a nonlinear transformation shown by (5) (see below). Previous investigations [2], [9] have indicated that the nonlinear transformation of the Masreliez type that transforms the prediction error $\varepsilon(k) = e(k) + v(k)$ to produce the optimal estimate of $e(k)$ is given by

$$\widehat{e}(k) = \rho[\varepsilon(k)] = \varepsilon(k) - \tanh\left[\frac{\varepsilon(k)}{\sigma_k^2}\right] \tag{5}$$

where $e(k)$ represents the prediction error in the absence $v(k)$ and $\sigma_k^2 = E[\varepsilon^2(k)] - 1$.

III. Adaptive Nonlinear QRD-LSL Interpolators for NBI Suppression

A. QRD-LSL Interpolators

When the $(p,f)^{th}$ order linear interpolation is used to achieve interference rejection, $\widehat{i}(j)$ is interpolated from p past and f future neighboring interference estimates, viz.,

$$\widehat{i}_{p,f}(j) = -\sum_{\substack{l=-p \\ l\neq 0}}^{f} b_{(p,f),l}(k-f)\,\widehat{i}(j+l), \quad 1-f \le j \le k-f \tag{6}$$

where $b_{(p,f),l}(k-f)$ is the interpolation coefficient at time k which remains fixed during the observation interval $1-f \le j \le k-f$ with the prewindowing condition on the interference estimate. By using (6), the $(p,f)^{th}$ order interpolation error at each time unit can be defined as

$$\begin{aligned} e_{p,f}^{I}(j) &= \widehat{i}(j) - \widehat{i}_{p,f}(j) = \widehat{i}(j) + \sum_{\substack{l=-p \\ l\neq 0}}^{f} b_{(p,f),l}(k-f)\,\widehat{i}(j+l), \\ 1-f &\le j \le k-f \end{aligned} \tag{7}$$

Notably, the interpolation error obtained in (7) will be used later to compute the interpolated interference estimate [see (15)]. If the most recent interference estimate used is $\widehat{i}(k)$, then the optimum interpolation coefficients in (7) can be determined by minimizing the sum of the $(p,f)^{th}$ order interpolation error squares, and consequently $O(M^2)$ operations are required. Yuan [7] developed *QRD-LSL interpolators* that require only $O(M)$ operations by utilizing a modified version of linear forward and backward predictions referred to as the intermediate forward and backward predictions. The QRD-LSL interpolators are implemented by combining the exact decoupling property of the LSL interpolators with the well-conditioned and numerically stable QR-decomposition technique. The QRD-LSL interpolation algorithm has been summarized in [7].

B. Adaptive Nonlinear ACM QRD-LSL Interpolators

We now develop an adaptive nonlinear QRD-LSL interpolator based on the ACM nonlinearity. A $(m+1)^{st}$ order predictor is used to compute the interference estimate $\widehat{i}(k) = \widehat{e}(k) + \overline{i}(k)$, where $\overline{i}(k) = \widehat{\mathbf{i}}_{m+1}^{T}(k-1)\,\mathbf{a}_{m+1}(k)$ is the output of the $(m+1)^{st}$-order linear predictor. The optimum tap-weight vector $\mathbf{a}_{m+1}(k) = [a_{m+1,1}(k), a_{m+1,2}(k), \cdots, a_{m+1,m+1}(k)]^T$ of the $(m+1)^{st}$ order predictor can be obtained by minimizing the sum of weighted forward prediction-error squares for $1 \le j \le k$, $\mathbf{E}^F = \sum_{j=1}^{k} \lambda^{k-j}\left[e_{m+1}^{F}(j)\right]^2$, where

$$\begin{aligned} e_{m+1}^{F}(j) &\triangleq \widehat{e}(j) = \widehat{i}(j) - \overline{i}(j) \\ &= \widehat{i}(j) - \mathbf{a}_{m+1}^{T}(k)\,\widehat{\mathbf{i}}_{m+1}(j-1), \quad 1 \le j \le k \end{aligned} \tag{8}$$

is the forward prediction error produced by the predictor at time j, in response to the tap-input vector $\widehat{\mathbf{i}}_{m+1}(j-1) = \left[\widehat{i}(j-1), \widehat{i}(j-2), \cdots, \widehat{i}(j-m-1)\right]^T$ and $0 \ll \lambda \le 1$ is the forgetting factor. The optimum value of the tap-weight vector, $\mathbf{a}_{m+1}(k)$, is defined by the following normal equations:

$$\mathbf{\Phi}_{m+1}(k-1)\,\mathbf{a}_{m+1}(k) = \boldsymbol{\theta}_{m+1}(k) \tag{9}$$

where $\mathbf{\Phi}_{m+1}(k-1)$ is the correlation matrix and $\boldsymbol{\theta}_{m+1}(k)$ is the cross-correlation vector. It is well known that a sequence of LS uncorrelated backward prediction errors at all instants of time forms a unique orthogonal basis set for the correlated input interference estimates, as shown by [8]

$$\widehat{\mathbf{i}}_{m+1}(j-1) = \mathbf{L}_{m+1}^{-1}(k-1)\,\mathbf{e}_{m+1}^{B}(j-1), \quad 1 \le j \le k \tag{10}$$

where $\mathbf{e}_{m+1}^{B}(j-1) = \left[e_0^B(j-1),\ e_1^B(j-1),\ \cdots,\ e_m^B(j-1)\right]^T$ and $\mathbf{L}_{m+1}(k-1)$ is the lower triangular transformation matrix. Since both $\mathbf{e}_{m+1}^{B}(j-1)$ and $\widehat{\mathbf{i}}_{m+1}(j-1)$ contains exactly the same information, the predicted value of $\widehat{i}(j)$ based on its $(m+1)$ previous interference estimates in $\widehat{\mathbf{i}}_{m+1}(j-1)$ can also be computed by using $\overline{i}(j) = \mathbf{a}_{m+1}^{T}(k)\widehat{\mathbf{i}}_{m+1}(j-1) = \mathbf{g}_{m+1}^{T}(k)\mathbf{e}_{m+1}^{B}(j-1)$ where $\mathbf{g}_{m+1}^{T}(k) = [g_0(k), g_1(k), \cdots, g_m(k)]$ is the regression coefficient vector at time k. Consequently, the QRD-LSL predictor for NBI suppression can also be implemented by minimizing $\mathbf{E}^F = \sum_{j=1}^{k} \lambda^{k-j}\left[\widehat{i}(j) - \mathbf{g}_{m+1}^{T}(k)\,\mathbf{e}_{m+1}^{B}(j-1)\right]^2$.

A relation between the $\mathbf{g}_{m+1}(k)$ and the $\mathbf{a}_{m+1}(k)$ can thus be written as

$$\mathbf{a}_{m+1}(k) = \mathbf{L}_{m+1}^{T}(k-1)\,\mathbf{g}_{m+1}(k) \tag{11}$$

The regression coefficient vector $\mathbf{g}_{m+1}(k)$ can then be computed by

$$g_m(k) = \frac{e_m^F(k) - e_{m+1}^F(k)}{e_m^B(k-1)}, \quad m = 0, 1, \cdots, M-1 \tag{12}$$

where $M = p + f$ is the final order of the QRD-LSL predictor. Meanwhile, the interpolation error $e_{p,f}^{I}(k-f)$ generated from the QRD-LSL interpolator can be used to compute the interpolated interference estimate $\bar{i}_{p,f}(k-f)$ with f units of delay, referred to herein as *interference interpolation*, by $\bar{i}_{p,f}(k-f) = \hat{i}(k-f) - e^I_{p,f}(k-f)$. Notably, $\bar{i}_{p,f}(k-f)$ is a more accurate version of the interference estimate than its prediction counterpart $\bar{i}_M(k) = \hat{i}(k) - e^F_M(k)$. Accordingly, a greater SNR improvement can be achieved by using interpolation as compared to that by prediction. Once the regression coefficients $g_m(k)$, $m = 0, 1, \cdots, M-1$ are computed, the "a priori" interference prediction, at time $(k+1)$, from the output of the QRD-LSL predictor can be computed by

$$\bar{i}_M(k+1) = \mathbf{g}^T_M(k)\,\mathbf{e}^B_M(k) = \sum_{i=0}^{M-1} g_i(k)\,e^B_i(k) \qquad (13)$$

where $\bar{i}_M(k+1)$ represents the interference prediction of $\hat{i}(k+1)$. The prediction error $\varepsilon_M(k)$ of order M can then be computed as

$$\varepsilon_M(k) = z(k) - \bar{i}_M(k) = z(k) - \left[\hat{i}(k) - e^F_M(k)\right] \qquad (14)$$

while the interpolation error of order (p,f) can be computed as

$$\begin{aligned} \varepsilon^I_{p,f}(k-f) &= z(k-f) - \bar{i}_{p,f}(k-f) \\ &= z(k-f) - \left[\hat{i}(k-f) - e^I_{p,f}(k-f)\right] \end{aligned} \qquad (15)$$

where both $e^F_M(k)$ and $e^I_{p,f}(k-f)$ are already computed by the QRD-LSL predictor and QRD-LSL interpolator, respectively. Notably, both $\varepsilon_M(k)$ and $\varepsilon^I_{p,f}(k-f)$ computed in (14) and (15) represent the estimates of the SS signal by employing the QRD-LSL predictor and QRD-LSL interpolator, respectively. They can be used to compute the SNR improvement which is a performance measure commonly used to verify the interference rejection filters. Figure 1 presents a signal-flow graph of an adaptive nonlinear lattice interpolator that employs the $(2,2)^{th}$ order QRD-LSL interpolator using the sequence BFBF.

IV. Simulation Results

Computer simulations are performed to assess the performance of the adaptive nonlinear QRD-LSL interpolator for unknown interference statistics. The rate of convergence and SNR improvement are compared using the nonlinear LMS-based predictor (NLMSP), nonlinear QRD-LSL predictor (NQRDP), and nonlinear QRD-LSL interpolator (NQRDI). The tap length is $M = 10$ for both predictors and $(p,f) = (5,5)$ for interpolator. The second-order AR jammer was obtained by passing white noise through a second-order IIR filter with both poles at $z = 0.99$, i.e, $i(k) = 1.98i(k-1) - 0.9801i(k-2) + u(k)$, where $\{u(k)\}$ is a white Gaussian process. The noise power was held constant at $\sigma^2_n = 0.01$, while the total of noise plus interference power was varied for input SNR range from -20 dB to -5 dB. The power of the SS signal was held constant with amplitudes ± 1. Figure 2 compares SNR improvements with a second-order AR jammer achieved by the three filters as a function of input SNR. The three filters are run for 4000 points and averages over 4000 independent trials are carried out. The SNR improvements are computed using the last 500 points. Figure 2 indicates that the NQRDI consistently provides a greater than around 4.5 dB SNR improvement compared to the NLMSP for input SNR range from -20 to -5 dB. This figure also reveals that the SNR improvement achieved by using the NQRDI with forgetting factor $\lambda = 0.965$ closely approaches the upper bound. Figure 3 compares the learning curves using the three filters with input SNR $= -20$ dB. The learning curves of the NLMSP, NQRDP, and NQRDI are generated by ensemble averaging the squared error versus the number of iterations n over 4000 independent trials of the experiment. According to Fig. 3, the NQRDP and NQRDI display a faster rate of convergence than that of the NLMSP.

Figure 4 compares the learning curves for a multiple tone sinusoidal interferer, $i(t) = A\cos(0.11t+\theta_1) + A\cos(0.12t+\theta_2) + A\cos(0.13t+\theta_3) + A\cos(0.14t+\theta_4) + A\cos(0.15t+\theta_5)$, where A is the amplitude of all sinusoidal interferences, chosen such that the input SNR $= -20$ dB and θ_1, θ_2, θ_3, θ_4, and θ_5 are random phases uniformly distributed from 0 to 2π. This figure illustrates that the rate of convergence of the nonlinear QRD-LSL algorithm is much faster than that of the nonlinear LMS algorithm in the presence of a multiple tone sinusoidal jammer. Figure 5 shows that tracking performance with time-variant jammers. In this simulation, we add 5 tones around $n = 500$. The figure shows that the NQRDI exhibits better tracking performance than its NQRDP and NLMSP counterparts in the presence of time-variant jammers.

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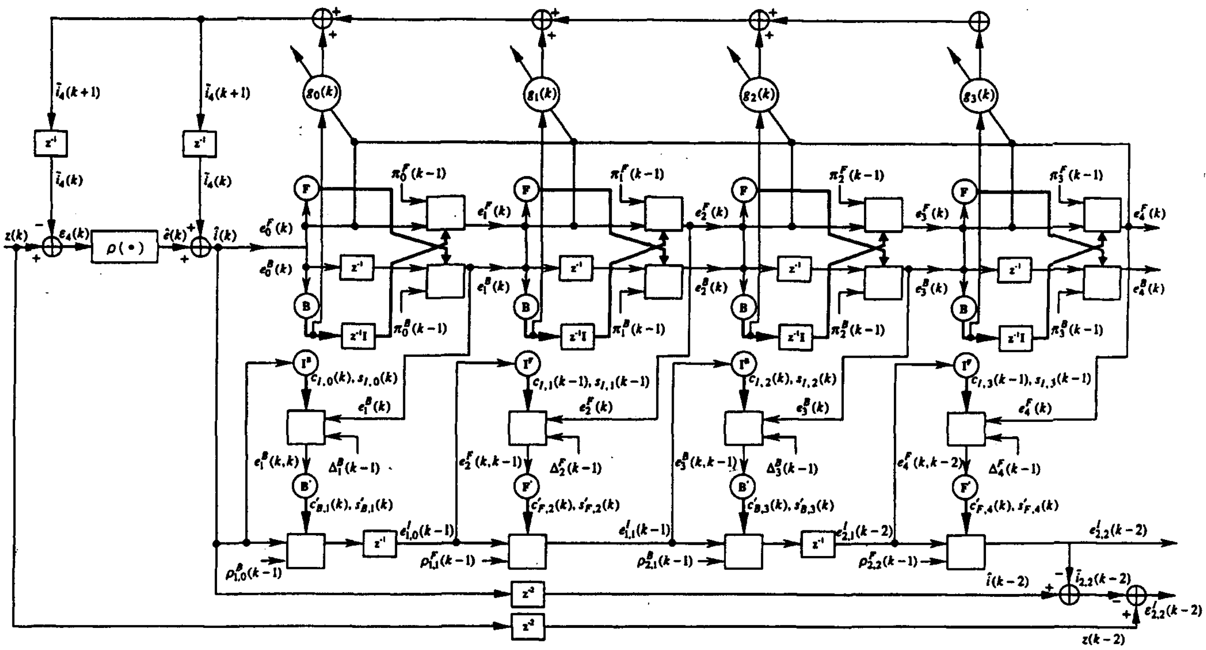

Fig. 1. Adaptive nonlinear QRD-LSL interpolator using sequence BFBF.

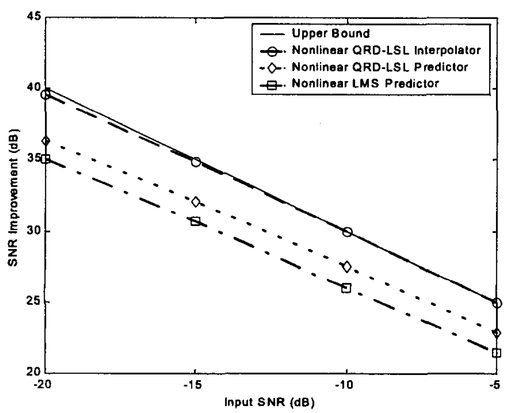

Fig. 2. Filter performance as a function of input SNR.

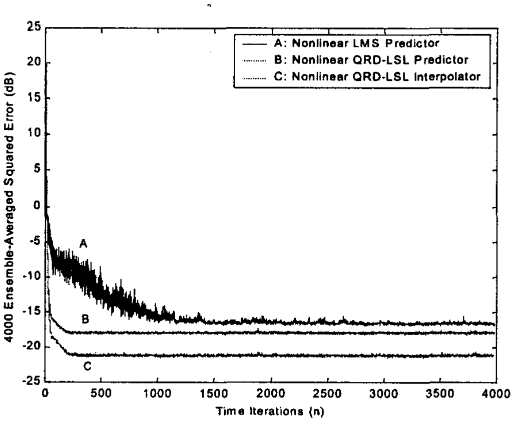

Fig. 3. Learning curves of a second-order AR jammer.

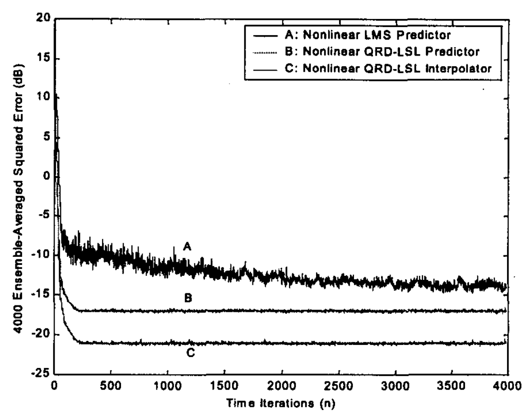

Fig. 4. Learning curves of a multiple tone jammer.

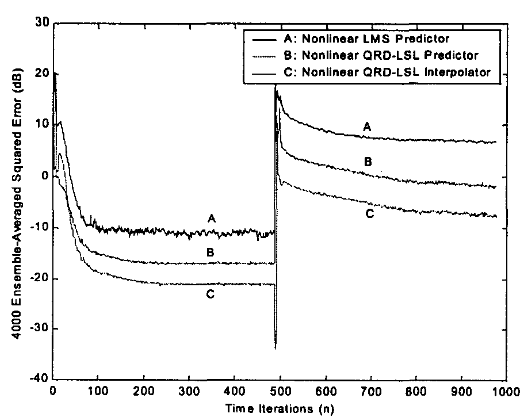

Fig. 5. Tracking performance in time-variant jammers.