Interpolation Lattice Filters for Narrowband Interference Cancellation in PN Spread Spectrum Communication Systems

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Abstract — **Previous work on the adaptive lattice filters for narrowband interference rejection in a direct sequence (DS) spread spectrum communication system has centered around the use of one-sided prediction only. For two-sided interpolation, that is known to achieve better interference rejection than its one-sided counterpart, only the least-mean-squared (LMS) algorithms have been considered thus far in the literature. However, the LMS algorithm has an extremely slow convergence rate. Consequently, the two-sided interpolation filters using the LMS algorithm may not be applicable for suppression of narrowband interference with a time-variant frequency. In this paper, an order-recursive QR-decomposition-based least squares lattice (QRD-LSL) interpolation filter is presented as a solution to the narrowband interference rejection in a DS spread spectrum system. Computer simulation results show that the QRD-LSL interpolation filters can achieve excellent performance in terms of convergence speed as well as SNR improvement for a multiple tone interference with random phase and second-order autoregressive (AR) interference.**

I. Introduction

The digital whitening technique has been used to enhance the spread spectrum communications performance in the presence of narrowband jamming and interference. This is due to the fact that the interference is generally assumed to be nonwhite and is therefore can be predicted by use of whitening techniques that can be implemented as a transversal filter (or tapped-delay-line realizations) [1]-[11]. On the other hand, both the spread signal and the thermal noise are wide-band processes and hence they cannot be predicted (or interpolated) accurately by using the neighboring samples. A one-sided prediction filter (or two-sided interpolation filter) is thus used to suppress the interference in order to improve the performance of a direct sequence (DS) spread spectrum. It has been shown in [3], [4], [12], and [13] that better interference rejection may be obtained by using the interpolation filter rather than the prediction filter with the same number of taps. This is due to the fact that the former makes better use of the correlation between the nearest neighboring samples than the latter.

Many articles have been published on the use of prediction filter employing the lattice structure for suppression of narrowband interference [14]-[20]. The lattice structure is known to have the advantages of modularity, robustness, and better numerical conditioning compared to the transversal structure [21]-[23]. Simulation results and experimental results in [14]-[20] indicate that the lattice structure has great potential in narrowband interference suppression especially in complex jamming environments (e.g., multiple jammers). However, to our knowledge, previous references to the use of lattice filters for suppression of narrowband interference have been primarily concerned with *prediction* only. Furthermore, even for one-sided prediction lattice filters considered in the available literature, except for [20] in which *least squares lattice (LSL)* has been considered, almost all the previous references use the *gradient adaptive lattice (GAL)*. The convergence behavior of the GAL algorithm is more rapid than that of the LMS algorithm, but inferior to that of *exact* recursive LSL algorithms [21].

This paper proposes the use of the order-recursive *QR-decomposition-based least squares lattice (QRD-LSL) interpolation* filters developed in [24] for suppression of narrowband interference in the DS spread spectrum system. As is well known, the QRD-LSL filter is the most important order-recursive adaptive filter since it represents the most fundamental form of an order-recursive adaptive filter. It is also well known that the performance of the QRD-LSL algorithm in a limited-precision environment is always superior to that of recursive LSL algorithms due to the fact that the QRD-LSL algorithm is a "square-root" type algorithm and, consequently, this results in a significantly reduced dynamic range of data [21]. The QRD-LSL algorithm combines highly desirable features of recursive least-squares estimation, QR-decomposition, and lattice structure and therefore it offers fast rate of convergence, good numerical properties, high level of computational efficiency, and modularity. In this paper, the performance of the QRD-LSL interpolation filter is evaluated for the rejection of narrowband interference when the narrowband interference is modeled as multiple-tone sinusoidal signals and a second-order autoregressive (AR) process respectively. We show the rate of convergence of the QRD-LSL interpolation filter is much faster than that of the LMS interpolation filter in both simulations. Furthermore, the former can achieve better SNR improvement than that of the latter.

II. System Model

The low-pass equivalent of a DS spread spectrum modulation waveform is given by [10]

$$m(t)=\sum_{k=0}^{L-1}c_k q(t-k\tau_c) \qquad (1)$$

where L is the number of pseudonoise (PN) chips per bit, τ_c is the chip interval, $\{c_k\}$ is the PN chip sequence used to spread the transmitted signal, and $q(t)$ is a rectangular pulse of

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duration τ_c. The transmitted signal is expressed as

$$s(t) = \sum_k b_k m(t - kT_b) \tag{2}$$

where $\{b_k\}$ is the binary information sequence and $T_b = L\tau_c$ is the bit duration. The received signal is then of the form

$$z(t) = s(t) + w(t) + i(t) \tag{3}$$

where $w(t)$ is wideband Gaussian noise, and $i(t)$ is narrowband interference. After the received signal has been processed by a matched filter and sampled at the chip rate $1/\tau_c$, the received signal becomes

$$z(n) = s(n) + w(n) + i(n) \tag{4}$$

where $\{s(n)\}$, $\{w(n)\}$, and $\{i(n)\}$ are the discrete-time sequences due to $\{s(t)\}$, $\{w(t)\}$, and $\{i(t)\}$, respectively and $\{s(n)\}$, $\{w(n)\}$, and $\{i(n)\}$ are assumed to be mutually independent. If the PN sequence is assumed to be truly random, the sequence $\{s(n)\}$ can be considered to be a sequence of i.i.d. binary random variables taking on values +1 or −1 with equal probability.

III. QRD-LSL Interpolation Filters

A. Least-Squares Interpolation Filters of Order (p, f)

An estimate of the interference $i(n)$ can be formed from the received signal $z(n)$ in (4). When using the $(p,f)^{th}$ order linear interpolation, we linearly estimate the present interference $i(i)$ from p past and f future neighboring received signal samples, viz.,

$$\hat{i}_{p,f}(i) \cong \hat{z}_{p,f}(i) = -\sum_{k=-p,k\neq 0}^{f} b_{(p,f),k}(n-f)z(i+k), \quad 1-f \le i \le n-f \tag{5}$$

where $b_{(p,f),k}(n\text{-}f)$ is the interpolation coefficient at time n which remains fixed during the observation interval $1\text{-}f \le i \le n\text{-}f$ with the prewindowing condition on the received signal samples. The reason that $\hat{i}_{p,f}(i) \cong \hat{z}_{p,f}(i)$ is because the spread signal $s(i)$ and the thermal noise $w(i)$ are wideband processes and hence they cannot be interpolated accurately using the neighboring samples, whereas the interference $i(i)$ is generally assumed to be nonwhite and therefore can be accurately interpolated. The length of the signal, n, is variable. The order, $N = p+f$. By using (5), the $(p,f)^{th}$ order interpolation error at each time unit can be defined as

$$\begin{aligned} e^{I}_{p,f}(i) = \hat{s}(i) &= z(i) - \hat{z}_{p,f}(i) \cong z(i) - \hat{i}_{p,f}(i) \\ &= z(i) + \sum_{k=-p,\,k\neq 0}^{f} b_{(p,f),k}(n-f)z(i+k), \quad 1-f \le i \le n-f \end{aligned} \tag{6}$$

Herein, we refer to any N^{th} order interpolation filter that operates on the present signal sample as well as p past and f future signal samples to produce the $(p,f)^{th}$ order interpolation error at its output as a $(p,f)^{th}$ order interpolation filter where $N = p+f$ is assumed implicitly. If the most recent signal sample used is $z(n)$, then (6) can be written in a matrix form for $1\text{-}f \le i \le n\text{-}f$ as (7) at the bottom of the next column. The optimum interpolation coefficients in (7) can be determined by minimizing the sum of the $(p,f)^{th}$ order interpolation error squares, $\sum_{i=1-f}^{n-f} [e^{I}_{p,f}(i)]^2$, and consequently $O(N^2)$ operations are required.

B. Order-Recursive LSL Interpolation Filters

Order-recursive LSL interpolation filters that require only $O(N)$ operations have been developed in [24] by introducing a modified version of linear forward and backward predictions referred to as the *intermediate forward and backward predictions*. Consider a sequence of n received signal samples, $\{z(i)\}$, $1 \le i \le n$. We refer to $z(i)$ as the most recent signal sample used for linear interpolation and refer to $z(i\text{-}f)$ as the present signal sample that will be estimated by its p past and f future neighboring signal samples, where $0 \le f \le N$ and $N = p + f$. The N^{th} order intermediate forward predictor $\hat{z}_N(i, i-f)$ is defined as the best possible predictor of $z(i)$ based on a weighted linear combination of the N previous signal samples without, however, taking the present data sample $z(i\text{-}f)$ into consideration, viz.,

$$\hat{z}_N(i, i-f) = -\sum_{k=1,\,k\neq f}^{N} a_{Nk}(n)z(i-k)$$

where $a_{Nk}(n)$, k=1, 2, ..., f-1, f+1, ..., N, are N^{th} order intermediate forward prediction coefficients which are optimized in the least-squares sense over the entire observation interval $1 \le i \le n$. The corresponding N^{th} order intermediate forward prediction error can therefore be written as

$$e^{F}_{N}(i, i-f) = z(i) - \hat{z}_N(i, i-f) = z(i) + \sum_{k=1,\,k\neq f}^{N} a_{Nk}(n)z(i-k), \quad 1 \le i \le n$$

The N^{th} order intermediate backward prediction error can similarly be found to be

$$e^{B}_{N}(i, i-f) = z(i-N) + \sum_{k=1,\,k\neq p}^{N} c_{Nk}(n)z(i+k-N)$$

where $c_{Nk}(n)$, k=1, 2, ..., p-1, p+1, ..., N, are N^{th} order intermediate backward prediction coefficients. As will be shown later, the intermediate forward and backward predictions form a bridge between linear predictions and linear interpolations.

We now use the principle of orthogonality to derive two basic equations that characterize the order-recursive LSL interpolation filter as one additional past signal sample and one additional future signal sample are taken into account respectively to estimate the present signal sample. We first derive the order-updated interpolation error as one additional *past* signal sample is considered. Consider the $(p,f)^{th}$ order interpolation in which we estimate the present signal sample $z(i\text{-}f)$ by $z(i\text{-}N)$, ..., $z(i\text{-}f\text{-}1)$, $z(i\text{-}f\text{+}1)$, ..., $z(i)$, $1 \le i \le n$. The corresponding $(p,f)^{th}$ order interpolation error is $e^{I}_{p,f}(i-f)$. The order-updated interpolation error, $e^{I}_{p+1,f}(i-f)$, requires knowledge of the additional past signal sample, $z(i\text{-}N\text{-}1)$, that

$$\begin{bmatrix} e^{I}_{p,f}(1-f) \\ \vdots \\ e^{I}_{p,f}(-1) \\ e^{I}_{p,f}(0) \\ e^{I}_{p,f}(1) \\ \vdots \\ e^{I}_{p,f}(n-f) \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ z(1) \\ \vdots \\ z(n-f-1) \\ z(n-f) \end{bmatrix} - \begin{bmatrix} z(1) & \cdots & 0 & 0 & \cdots & 0 \\ z(2) & & \vdots & \vdots & & \vdots \\ z(3) & \cdots & 0 & 0 & \cdots & 0 \\ & \cdots & z(1) & z(1) & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ & \cdots & & & \cdots & \\ \vdots & & \vdots & \vdots & & \vdots \\ z(n) & \cdots & z(n-f+1) & z(n-f-1) & \cdots & z(n-N) \end{bmatrix} \begin{bmatrix} -b_{(p,f),f}(n-f) \\ -b_{(p,f),f-1}(n-f) \\ \vdots \\ -b_{(p,f),1}(n-f) \\ -b_{(p,f),-1}(n-f) \\ \vdots \\ -b_{(p,f),-p}(n-f) \end{bmatrix} \tag{7}$$

can be obtained by the $(N+1)^{st}$ order intermediate backward prediction error, $e^B_{N+1}(i,i-f)$. If therefore we were to compute $e^B_{N+1}(i,i-f)$, the additional past signal sample $z(i-N-1)$ needed for computing $e^I_{p+1,f}(i-f)$ would be found in the composition of the $e^B_{N+1}(i,i-f)$. Thus, if we decompose $e^I_{p,f}(i-f)$ into two mutually orthogonal parts in which the first part is interpolable by the input $e^B_{N+1}(i,i-f)$ [i.e., treat $e^B_{N+1}(i,i-f)$ as the same amount of information carried by the additional past signal sample $z(i-N-1)$ used to estimate $z(i-f)$ in the LSL interpolation filter] and the second part is the residual [i.e., the part that is not interpolable by $z(i-N-1)$], $e^I_{p+1,f}(i-f)$, which is orthogonal to $e^B_{N+1}(i,i-f)$, resulting from the LS estimation, we may thus write

$$e^I_{p+1,f}(i-f) = e^I_{p,f}(i-f) - k^B_{p+1,f}(n) e^B_{N+1}(i,i-f), \quad i=1,2,\ldots,n \tag{8}$$

where $k^B_{p+1,f}(n)$ is the coefficient that can be determined by applying the principle of orthogonality such that

$$\sum_{i=1}^{n} e^I_{p+1,f}(i-f) e^B_{N+1}(i,i-f) = 0 \tag{9}$$

The computation of the order-updated interpolation error of the present signal sample as one additional *future* signal sample $z(i)$ is considered can similarly be derived to be

$$e^I_{p,f+1}(i-f-1) = e^I_{p,f}(i-f-1) - k^F_{p,f+1}(n) e^F_{N+1}(i,i-f-1) \quad i=1,2,\cdots,n \tag{10}$$

To construct an LSL interpolation filter of order (p,f), equations (8) and (10) must be applied p and f times, respectively. However, any sequencing between these two equations is permissible. Consequently, there are $C^p_N = C^f_N = N!/p!f!$ permissible realizations for an LSL interpolation filter of order (p,f). For example, to construct a $(2,2)^{th}$ order interpolation filter, we have a total of permissible realizations which may be identified by the sequences BFBF, FBFB, BBFF, FFBB, FBBF, and BFFB of intermediate backward (B) and intermediate forward (F) prediction errors used in (8) and (10) respectively. The six permissible sequences, in fact, form six possible orthogonal basis sets that can be used to implement a $(2,2)^{th}$ order LSL interpolation filter.

Equations (8) and (10) reveal that both intermediate forward and backward prediction errors are needed in the computation of the order-updated interpolation error. However, unlike the standard forward and backward prediction errors that are directly accessible from an LSL prediction filter, the intermediate forward and backward prediction errors at this point are still unknown. They, however, can be computed through the following relations. Consider the $(N+1)^{st}$ order intermediate backward prediction in which we predict the signal sample $z(i-N-1)$ by using $z(i-N)$, ..., $z(i-f-1)$, $z(i-f+1)$, ..., $z(i)$. The resulting intermediate backward prediction error is $e^B_{N+1}(i,i-f)$. The $(N+1)^{st}$ order backward prediction error, $e^B_{N+1}(i)$, however, can be generated by predicting $z(i-N-1)$ by using $z(i-N)$, ..., $z(i)$, which requires knowledge of the present signal sample, $z(i-f)$, that can be obtained by the $(p,f)^{th}$ order interpolation error, $e^I_{p,f}(i-f)$. If therefore we were to compute $e^I_{p,f}(i-f)$, the present signal sample $z(i-f)$ needed for computing $e^B_{N+1}(i)$ would be found in the composition of the $e^I_{p,f}(i-f)$. Thus, if we decompose $e^B_{N+1}(i,i-f)$ into two mutually orthogonal parts in which the first part is predictable by the input $z(i-f)$ and the second part is the residual, $e^B_{N+1}(i)$, which is orthogonal to $e^I_{p,f}(i-f)$, resulting from the LS estimation. We may thus write

$$e^B_{N+1}(i,i-f) = e^B_{N+1}(i) + l^B_{N+1}(n) e^I_{p,f}(i-f), \quad i=1,2,\cdots,n \tag{11}$$

where the coefficient $l^B_{N+1}(n)$ can be determined by applying the principle of orthogonality. Note that $e^B_{N+1}(i)$ is directly accessible from an $(N+1)^{st}$ order LSL prediction filter that can be embedded into an LSL interpolation filter of order $(p+1,f)$ and $e^I_{p,f}(i-f)$ is already computed from the previous interpolation lattice stage of the LSL interpolation filter. It can be similarly shown that

$$e^F_{N+1}(i,i-f-1) = e^F_{N+1}(i) + l^F_{N+1}(n) e^I_{p,f}(i-f-1), \quad i=1,2,\ldots,n \tag{12}$$

where $e^F_{N+1}(i)$ is directly accessible from an $(N+1)^{st}$ order LSL prediction filter that can be embedded into an LSL interpolation filter of order $(p,f+1)$ and $e^I_{p,f}(i-f-1)$ is already computed from the previous interpolation lattice stage. The relations shown in (11) and (12) can be used to compute the intermediate backward and forward prediction errors respectively and they connect together the linear predictions, intermediate predictions, and linear interpolations. In fact, (8), (10), (11), and (12), together with the LSL prediction filter [21], constitute an order-recursive LSL interpolation filter [24].

C. Exact Decoupling Property of LSL Interpolation Filters

When a sequence of p past and f future signal samples is taken into consideration to estimate the present signal sample $z(n-f)$, appropriate combinations of f delayed intermediate forward prediction errors and p delayed intermediate backward prediction errors, followed finally by the interpolation error, $e^I_{p,f}(i-f)$, form C^f_N sets of LS orthogonal bases [24]. Each of the C^f_N orthogonal basis sets can be shown to provide an orthogonal basis for [$z(n-N)$, ..., $z(n-f-1)$, $z(n-f)$, $z(n-f+1)$, ..., $z(n)$], spanned by the present signal sample, $z(n-f)$, p past signal samples, and f future signal samples. The LS orthogonality among all the elements within each of these orthogonal bases is referred to as the exact decoupling property of the LSL interpolation filters. Owing to the exact decoupling property, a $(p,f)^{th}$ order LSL interpolation filter automatically generates all N of the outputs that would be provided by N separate transversal interpolation filters of length 1, 2, ..., N, (i.e., a multiple order filter) where $N = p+f$. Higher order lattice filters are obtained from lower order ones by simply adding more stages, leaving the original stages unchanged. This modular structure permits dynamic assignment, and rapid automatic determination of the most effective filter length. Consequently, optimum removal of strong narrowband interference of unknown or time-varying bandwidth may be achieved. The QRD-LSL interpolation filters are implemented by combining the exact decoupling property of the LSL interpolation filters with the well-conditioned and numerically stable QR-decomposition technique. The QRD-LSL interpolation algorithm has been summarized in [24].

IV. Simulation Results

Computer simulations have been carried out to compare the learning curves of the LMS interpolation filter and QRD-LSL interpolation filter for multiple tone interference with random phase and a second-order AR interference. In all the simulations, the number of taps used for the LMS prediction filters was 10 and the order used for the LMS interpolation filters was $(p,f)=(5,5)$ whereas the order used for the QRD-LSL filters for prediction and interpolation was $(p,f)=(10,0)$ and $(p,f)=(5,5)$ respectively. The forgetting factor, λ, used in all the QRD-LSL simulations is set to be equal to 0.99. The step-size parameter, μ, used in all the LMS simulations is set to be equal to 0.00003. The reason we choose the LMS interpolation filter for comparison is because only the LMS algorithm for two-sided interpolation filter has been considered thus far in the literature for narrowband interference suppression. All the adaptive lattice filters for narrowband interference rejection up to this point have been concerned with prediction only.

Each learning curve was obtained by ensemble-averaging the squared value of $[s(n)-\hat{s}(n)]$ over 200 independent trials of the experiment, where $\hat{s}(n)$ is the output of the LMS interpolation (or prediction) filter and QRD-LSL interpolation (or prediction) filter with input $z(n)$. The power of the wideband Gaussian noise $w(n)$ is kept constant at $\sigma_w^2 = 0.01$. Fig. 1 shows results for a multiple tone sinusoidal interferer having frequency of 0.15 radians, 0.4 radians and 0.65 radians respectively, i.e., $i(t) = A\cos(0.15t+\theta_1) + A\cos(0.4t+\theta_2) + A\cos(0.65t+\theta_3)$, where A is the amplitude of all sinusoidal interferences that is chosen such that the jammer-to-signal power ratio (J/S) is 20dB and θ_1, θ_2, and θ_3 are random phases uniformly distributed from 0 to 2π. This figure shows that the rate of convergence of the QRD-LSL algorithm is much faster than that of the LMS algorithm in the presence of a multiple tone sinusoidal jammer. And the steady-state mean square error (MSE) of QRD-LSL interpolation filter is also better than the LMS interpolation filter. This implies that the average SNR improvement by using QRD-LSL interpolation filter is more efficient for a multiple tone sinusoidal jammer. Fig. 2 shows results for a second-order AR jammer that is obtained by passing white noise through a second-order IIR filter with two poles at $0.9557+j0.1914$ and $0.9557-j0.1914$, i.e., $i(n) = 1.9114i(n-1)-0.95i(n-2)+e(n)$, where $\{e(n)\}$ is white Gaussian noise. The QRD-LSL interpolation filter shows much faster convergence and smaller steady-state MSE as compared with the LMS interpolation filter in the presence of an AR interference. The average SNR improvement over 500 data points was calculated and shown in Table 1. The result shows that the average SNR improvement achieved by using the QRD-LSL interpolation filter is better than that by using the LMS interpolation filter.

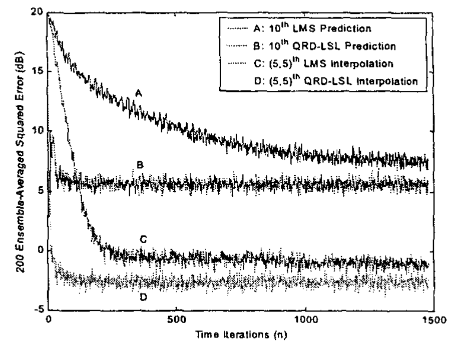

Figure 1. Multiple tone jammers with random phase (forgetting factor $\lambda = 0.99$ and step-size parameter $\mu = 0.00003$)

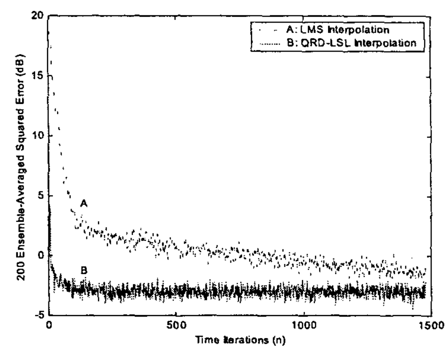

Figure 2. Second-order AR jammer

	SNR improvement (dB)	
	Multiple-tone	AR
LMS prediction	12.28	14.78
QRD-LSL prediction	14.45	17.25
LMS interpolation	21.35	21.28
QRD-LSL interpolation	22.66	23.24

Table 1. SNR improvement for multiple-tone and AR jammers

V. Conclusions

The use of the LMS algorithm results in slow convergence. We could have used the well-known RLS algorithm to achieve interference rejection. The convergence of the RLS algorithm is an order of magnitude faster than the LMS algorithm. Consequently, the RLS algorithm has excellent tracking capabilities in a time-varying environment. However, the computational operations required by the RLS algorithm are $O(N^2)$. Moreover, the RLS algorithm tends to accumulate finite precision errors that eventually cause instability of the adaptive system because it involves inversion of a data correlation matrix [25], [26]. In this paper we use the well-known rotation-based QRD-LSL algorithm for overcoming the difficulties associated with the LMS algorithm and the RLS algorithm. The QRD-LSL interpolation filters are developed by combining the exact decoupling property with the well-conditioned and numerically stable QR-decomposition technique. The advantages of using the QRD-LSL interpolation filters include fast rate of convergence, excellent numerical properties, low sensitivity to round off error and parameter perturbation, high level of computational efficiency, order-recursive property, and high degrees of parallelism. The QRD-LSL interpolation filters can also be modified as a nonlinear interpolation lattice filter corresponding to the nonlinear transversal interpolation filters developed in [13].

VI. References

[1] F. M. Hsu and A. A. Giordano, "Digital whitening techniques for improving spread-spectrum communications performance in the presence of narrow-band jamming and interference," *IEEE Trans. Commun.*, vol. COM-26, pp. 209-216, Feb. 1978.

[2] J. W. Ketchum and J. G. Proakis, "Adaptive algorithms for estimating and suppressing narrow-band interference in PN spread-spectrum systems," *IEEE Trans. Commun.*, vol. COM-30, pp. 913-924, May 1982.

[3] L. Li, and L. B. Milstein, "Rejection of narrow-band interference in PN spread-spectrum systems using transversal filters," *IEEE Trans. Commun.*, vol. COM-30, pp. 925-928, May 1982.

[4] L. Li, and L. B. Milstein, "Rejection of pulsed CW interference in PN spread-spectrum systems using complex adaptive filters," *IEEE Trans. Commun.*, vol. COM-31, pp. 10-20, Jan. 1983.

[5] L. B. Milstein, "Interference rejection techniques in spread spectrum communications," *Proceedings of the IEEE*, vol. 76, no. 6, pp. 657-671, June 1988.

[6] E. Masry, "Closed-form analytical results for the rejection of narrow-band interference in PN spread-spectrum systems - Part I: Linear prediction filters," *IEEE Trans. Commun.*, vol. COM-32, pp. 888-896, Aug. 1984.

[7] R. A. Iltis and L. B. Milstein, "Performance analysis of narrow-band interference rejection techniques in DS spread-spectrum systems," *IEEE Trans. Commun.*, vol. COM-32, pp. 1169-1177, Nov. 1984.

[8] R. A. Iltis and L. B. Milstein, "An approximate statistical analysis of the Widrow LMS algorithm with application to narrow-band interference rejection," *IEEE Trans. Commun.*, vol. COM-33, pp. 121-130, Feb. 1985.

[9] G. J. Saulnier, P. K. Das, and L. B. Milstein, "An adaptive digital suppression filter for direct-sequence spread-spectrum communications," *IEEE J. Select. Areas Commun.*, vol. SAC-3, no. 5, September 1985.

[10] R. Vijayan and H. V. Poor, "Nonlinear techniques for interference suppression in spread-spectrum systems," *IEEE Trans. Commun.*, vol. COM-38, pp. 1060-1065, July 1990.

[11] W. R. Wu and F. F. Yu, "New nonlinear algorithms for estimating and suppressing narrowband interference in DS spread spectrum systems," *IEEE Trans. Commun.*, vol. COM-44, pp. 508-515, April 1996.

[12] E. Masry, "Closed-form analytical results for the rejection of narrow-band interference in PN spread-spectrum systems - Part II: Linear interpolation filters," *IEEE Trans. Commun.*, vol. COM-33, pp. 10-19, January 1985.

[13] L. A. Rusch and H. V. Poor, "Narrowband interference suppression in CDMA spread spectrum communications," *IEEE Trans. Commun.*, vol. COM-42, pp. 1969-1979, February/March/April 1994.

[14] J. Guilford and P. Das, "The use of the adaptive lattice filter for narrowband jammer rejection in DS spread spectrum systems," *Proc. IEEE Int. Conf. on Commun.*, pp. 822-826, June 22-26, 1985.

[15] G. J. Saulnier, K. Yum, and P. Das, "The suppression of tone jammers using adaptive lattice filtering," *Proc. IEEE Int. Conf. on Commun.*, pp. 24.4.1-24.4.5, June 1987.

[16] Y. He, S.-F. Lei, P. Das, and G. J. Saulnier, "Suppression of narrowband jammers in a DS spread spectrum receiver using modified adaptive filtering technique," *Proc. Globecom '88*, vol. 1, pp. 540-545, 1988.

[17] J. R. Zeidler, K. C. Chew, R. A. Axford, and W. H. Ku, "Frequency tracking performance of adaptive lattice filters," *Proc. 25th Asilomar Conf. on Signals, Systems and Computers*, vol. 2, pp. 643-649, 1991.

[18] F. Takawira, "Adaptive lattice filters for narrowband interference rejection in DS spread spectrum systems," *Proc. 1994 IEEE South Africa Symp. on Commun. and Signal Processing*, pp. 1-5, 1994.

[19] G. J. Saulnier, W. A. Haskins, and P. Das, "Tone jammer suppression in a direct sequence spread spectrum receiver using adaptive lattice and transversal filters," *Proc. MILCOM'87*, pp. 123-127, 1987.

[20] W. S. Hodgkiss, Jr., and J. A. Presley, Jr., "Adaptive tracking of multiple sinusoids whose power levels are widely separated," *IEEE Trans. Circuits and Systems*, vol. CAS-28, no. 6, pp. 550-561, June 1981.

[21] S. Haykin, *Adaptive Filter Theory*, Englewood Cliffs, New Jersey: Prentice-Hall Inc., 1996, 3rd ed.

[22] B. Friedlander, "Lattice Filters for Adaptive Processing," *Proceedings of the IEEE*, vol. 70, no. 8, pp. 829-867, August 1982.

[23] Peter Strobach, *Linear Prediction Theory - A Mathematical Basis for Adaptive Systems*, Springer - Verlag, 1990.

[24] J. T. Yuan, "QR-decomposition-based least-squares lattice interpolators," *IEEE Trans. Signal Processing*, vol. 48, no. 1, pp. 70-79, Jan. 2000.

[25] D. F. Marshall and W. K. Jenkins, "A fast quasi-Newton adaptive filtering algorithm," *IEEE Trans. Signal Processing*, vol. 40, no. 7, pp. 1652-1662, July 1992.

[26] H. V. Poor and X. Wang, "Code-aided interference suppression for DS/CDMA communications - part II: parallel blind adaptive implementations," *IEEE Trans. Commun.*, vol. COM-45, no. 9, pp. 1112-1122, September 1997.