Correspondence

Asymmetric Interpolation Lattice

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Abstract—**This paper presents a new lattice structure for linear interpolation. The interpolation lattice structure is *asymmetric* in the sense that the number of past and future values linearly weighted to estimate the current value does not have to be identical. The lattice structure provides a computationally efficient and structurally flexible realization for the interpolation lattice. It also leads to a generalization of the concepts of the well-known linear prediction lattice and symmetric interpolation lattice.**

I. Introduction

Linear interpolation has many applications in signal processing. Some well-known theoretical properties and results in linear interpolation were discussed in [1]–[3]. It is also well known that lattice realizations of finite impulse response (FIR) filters offer advantages over tapped-delay-line realizations. A lattice structure for the symmetric interpolation filter was first developed in [4]. The asymmetric interpolation lattice structure developed in this paper generalizes the results in [2]–[4]. It also provides a computationally efficient and structurally flexible implementation for the interpolation lattice. The interpolation lattice is asymmetric in the sense that the number of past and future values linearly weighted to estimate the current value is not necessarily identical. It reduces to the well-known linear prediction lattice and symmetric interpolation lattice, respectively, when no future signal samples and when an equal number of past and future signal samples are used to estimate the current signal sample. The asymmetric interpolation lattice is developed both for stationary random processes using a minimum mean square error (MMSE) criterion in Section II and for prewindowed nonrandom data using a least-squares (LS) criterion in Section III. Learning curves comparing prediction and interpolation lattice performances are given in Section IV.

II. MMSE Interpolation Lattice

Let $x(n)$ denote a real, wide-sense stationary random process taken at the nth sample. The problem of (p, f)th order linear asymmetric interpolation can be defined as follows. An estimate of the current signal sample $x(n)$, denoted by $\hat{x}_{p,f}(n)$, can be obtained by linearly weighting p previous and f future signal samples

$$\hat{x}_{p,f}(n) = -\sum_{\substack{i=-p \\ i\neq 0}}^{f} b_{(p,f),i}\,x(n+i)$$

where $b_{(p,f),i}$ is the interpolation coefficient of order (p, f), and p and f are both positive integers. The (p, f)th order linear interpolation error $e^I_{p,f}(n)$ is therefore given by

$$e^I_{p,f}(n) = x(n) - \hat{x}_{p,f}(n) = \sum_{i=-p}^{f} b_{(p,f),i}\,x(n+i) = \mathbf{b}^T_{p,f}\mathbf{x}_{p,f}(n) \tag{1}$$

where $b_{(p,f),0}$ is defined as unity, $\mathbf{b}^T_{p,f} = [b_{(p,f),f}, \cdots, b_{(p,f),1}, 1, b_{(p,f),-1}, \cdots, b_{(p,f),-p}]$, and $\mathbf{x}^T_{p,f}(n) = [x(n+f), x(n+f-1), \cdots, x(n-p)]$. The superscript T denotes the transpose operation. We choose the interpolation coefficients in (1) to minimize the mean square interpolation error $E\{(e^I_{p,f}(n))^2\}$. The minimization can be accomplished by using the well-known orthogonality principle, which results in the augmented asymmetric interpolation normal equation

$$\mathbf{R}_q\mathbf{b}_{p,f} = \mathbf{i}_{p,f} \tag{2}$$

where $\mathbf{i}^T_{p,f} = [0, \cdots, 0, I_{p,f}, 0, \cdots, 0]$, and $q = p + f$. The matrix $\mathbf{R}_q$ in (2) is the qth order Toeplitz autocorrelation matrix of the signal sample $x(n)$ and $I_{p,f}$ in the vector $\mathbf{i}_{p,f}$ is the minimum mean square (MMS) interpolation error of order (p, f)

Our objective is to express the interpolation error $e^I_{p,f}(n)$ in (1) as a linear combination of the mutually orthogonal forward prediction errors $\{e^F_q(n+f), e^F_{q-1}(n+f-1), \cdots, e^F_0(n-p)\}$. We first linearly transform the signal samples $\{x(n+f), x(n+f-1), \cdots, x(n-p+1), x(n-p)\}$ to the forward prediction errors by

$$\mathbf{e}^F_q(n) = \mathbf{U}_q\mathbf{x}_{p,f}(n) \tag{3}$$

where $\mathbf{e}^{F^T}_q(n) = [e^F_q(n+f), e^F_{q-1}(n+f-1), \cdots, e^F_0(n-p)]$. The matrix $\mathbf{U}_q$ in (3) is the $(q+1)$-by-$(q+1)$ upper-triangular matrix with one's on the diagonal in the UL Cholesky factorization of the qth order autocorrelation matrix $\mathbf{R}_q$ [5, p. 221], given by the relation

$$\mathbf{R}^{-1}_q = \mathbf{U}^T_q(\mathbf{P}_q)^{-1}\mathbf{U}_q. \tag{4}$$

The matrix $\mathbf{P}_q$ in (4) is diagonal with the MMS prediction errors of orders $q, q-1, \cdots, 0$ on the main diagonal. The time-delayed (p, f)th order interpolation error $e^I_{p,f}(n-f)$ can be obtained by using (1), (2), (3), and (4) to be

$$\begin{aligned} &e^I_{p,f}(n-f) \\ &= I_{p,f}\left\{\left[\left(\frac{1}{P_p}\right)e^F_p(n-f)\right] + \left[\left(\frac{a_{p+1,1}}{P_{p+1}}\right)e^F_{p+1}(n-f+1)\right.\right. \\ &\quad \left.\left. + \cdots + \left(\frac{a_{q-1,f-1}}{P_{q-1}}\right)e^F_{q-1}(n-1) + \left(\frac{a_{q,f}}{P_q}\right)e^F_q(n)\right]\right\} \end{aligned} \tag{5}$$

where $a_{m,i}$ is the prediction coefficient of order m and P_m is the MMS prediction error of order m. Equation (5) has a nice interpretation. The pth order forward prediction error $e^F_p(n-f)$ in the first bracket represents the error of estimating signal sample $x(n-f)$ based on its p previous samples. The prediction error $e^F_p(n-f)$ can be improved by using f signal samples subsequent to the sample $x(n-f)$. These f signal samples corresponding to those f forward prediction errors in the second bracket of (5) can be thought of as a correction term adding some improvements to the prediction error $e^F_p(n-f)$. The (p, f)th-order MMS interpolation error $I_{p,f}$ can be obtained by realizing that all the forward prediction errors in (5) are mutually orthogonal as follows:

$$I_{p,f} = 1\bigg/\left(\frac{1}{P_p} + \frac{a^2_{p+1,1}}{P_{p+1}} + \cdots + \frac{a^2_{q-1,f-1}}{P_{q-1}} + \frac{a^2_{q,f}}{P_q}\right). \tag{6}$$

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The delayed interpolation error $e^I_{p,f}(n-f)$ can also be similarly expressed in terms of a linear combination of the mutually orthogonal backward prediction errors by using the LU Cholesky factorization

$$e^I_{p,f}(n-f) = I_{p,f}\left\{\left(\frac{1}{P_f}\right)e^B_f(n) + \left(\frac{a_{f+1,1}}{P_{f+1}}\right)e^B_{f+1}(n) + \cdots + \left(\frac{a_{q-1,p-1}}{P_{q-1}}\right)e^B_{q-1}(n) + \left(\frac{a_{q,p}}{P_q}\right)e^B_q(n)\right\} \quad (7)$$

where the (p,f)th order MMS interpolation error $I_{p,f}$ can also be found to be

$$I_{p,f} = 1\Big/\left(\frac{1}{P_f} + \frac{a^2_{f+1,1}}{P_{f+1}} + \cdots + \frac{a^2_{q-1,p-1}}{P_{q-1}} + \frac{a^2_{q,p}}{P_q}\right). \quad (8)$$

Both equation pairs (5) and (6) and (7) and (8) allow us to compute the interpolation error of order (p,f) *nonrecursively* from the results of the $(p+f)$th = qth order linear prediction. The results computed by using (5) and (7) will be exactly the same.

The derivation of the asymmetric *order-recursive* interpolation lattice solution can be performed as follows: Similar to the derivation of (5), the $(p,f+1)$th-order delayed interpolation error $e^I_{p,f+1}(n-f-1)$ can be expressed in terms of forward prediction errors as

$$\begin{aligned} &e^I_{p,f+1}(n-f-1) \\ &= I_{p,f+1}\left\{\left(\frac{a_{q+1,f+1}}{P_{q+1}}\right)e^F_{q+1}(n) + \left[\left(\frac{1}{P_p}\right)e^F_p(n-f-1) + \left(\frac{a_{p+1,1}}{P_{p+1}}\right)e^F_{p+1}(n-f) + \cdots + \left(\frac{a_{q,f}}{P_q}\right)e^F_q(n-1)\right]\right\} \end{aligned} \quad (9)$$

where $I_{p,f+1}$ is the MMS interpolation error of order $(p,f+1)$. If $I_{p,f+1}$ and $I_{p,f}$ are related as $I_{p,f+1} = \beta^F_{p,f+1}I_{p,f}$, using (5) we have

$$\begin{aligned} &e^I_{p,f+1}(n-f-1) \\ &= \beta^F_{p,f+1}\left\{e^I_{p,f}(n-f-1) + \left(\frac{a_{q+1,f+1}I_{p,f}}{P_{q+1}}\right)e^F_{q+1}(n)\right\} \end{aligned} \quad (10)$$

where $\beta^F_{p,f+1}$ is a constant. The quantity $e^I_{p,f+1}(n-f-1)$ in (10) is the $(p,f+1)$th-order interpolation error in estimating the signal sample $x(n-f-1)$ by using its p past signal samples $x(n-f-2),\cdots,x(n-q-1)$ and its $f+1$ future signal samples $x(n-f),\cdots,x(n)$. Note that $x(n)$ is the most recent signal sample being used to estimate the sample being interpolated. Also note that there is one *delay* involved in forming the new order-update interpolation error as each additional *future* signal sample is used. The constant $\beta^F_{p,f+1}$ can be obtained by using (10) to be

$$\beta^F_{p,f+1} = \frac{I_{p,f+1}}{I_{p,f}} = 1\Big/\left(1 + \frac{I_{p,f}a^2_{q+1,f+1}}{P_{q+1}}\right). \quad (11)$$

Note that $\beta^F_{p,f+1}$ has a value between zero and one. This implies that the MMS interpolation error will decrease as the length of the filter is increased. Equation (11) provides an order-update recursion for the MMS interpolation error as each additional future signal sample is weighted to estimate the present signal sample. In a similar manner, as one additional past signal sample $x(n-p-1)$ is linearly weighted to estimate the present signal sample $x(n)$, the delayed interpolation error of order $(p+1,f)$ can be expressed recursively as

$$\begin{aligned} &e^I_{p+1,f}(n-f) \\ &= \beta^B_{p+1,f}\left\{e^I_{p,f}(n-f) + \left(\frac{a_{q+1,p+1}I_{p,f}}{P_{q+1}}\right)e^B_{q+1}(n)\right\} \end{aligned} \quad (12)$$

where $\beta^B_{p+1,f}$ can be obtained to be

$$\beta^B_{p+1,f} = \frac{I_{p+1,f}}{I_{p,f}} = 1\Big/\left(1 + \frac{I_{p,f}a^2_{q+1,p+1}}{P_{q+1}}\right). \quad (13)$$

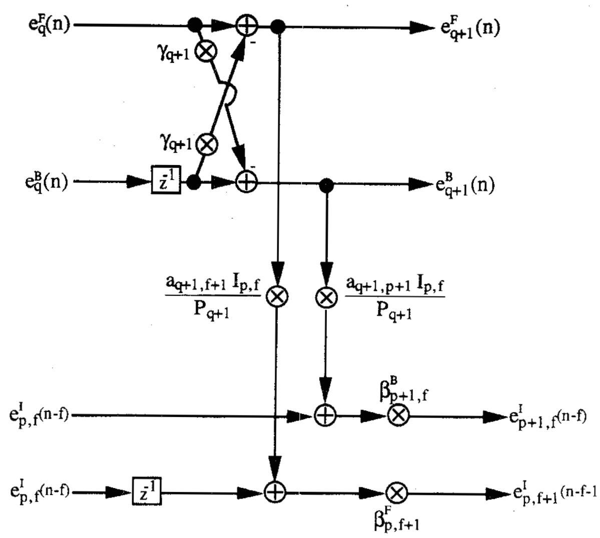

Fig. 1. Single-stage order-recursive lattice realization of the $(p,f+1)$st-order and the $(p+1,f)$th-order interpolation error filter.

Equation (13) provides an order update recursion for the MMS interpolation error as each additional *past* signal sample is weighted to estimate the present signal sample. Equations (10)–(13) provide *order-recursive* updates for the interpolation error. An illustration of this interpolation lattice solution is given in Fig. 1, which depicts a single-stage lattice realization of the $(p,f+1)$st and $(p+1,f)$th order-updated interpolation errors from the current (p,f)th order interpolation error. As illustrated in Fig. 1, (10) and (12) require that a higher order *forward* prediction error be used to update the current interpolation error as each additional *future* signal sample is used, and a higher order *backward* prediction error be used as each additional *past* signal sample is used. Hence, the use of a future signal sample corresponds to an updating of the interpolation error with a forward prediction error, and the use of a past signal sample corresponds to an updating of interpolation error by using a backward prediction error. A combined use of both (10) and (12) will show that an interpolation operation of any order (p,f) can be expressed in terms of *mutually orthogonal* forward and backward prediction errors [7]. An example is shown in Figs. 2 and 3.

Both order-recursive and nonrecursive interpolation lattice solutions of order (p,f) require the solution of the qth-order linear prediction, which can be obtained by using the Levinson algorithm. Note that in the symmetric interpolation case of order (p,p), with known $(2p)$th-order linear prediction results, only $O(p)$ operations are required compared to the $O(p^2)$ operations needed by the algorithm developed in [4]. For the more general (p,f)th order asymmetric interpolation, $O(\min(p,f))$ operations are required once the results of the qth order linear prediction are known. The asymmetric interpolation can also be viewed as a generalization of forward and backward predictions and symmetric interpolation by letting $(p,f)=(p,0),(p,f)=(0,f)$, and $(p,f)=(p,p)$, respectively.

The asymmetric interpolation error power of an AR process of order m can be obtained by using (6) and a well known property of the AR process $P_0 \ge P_1 \ge \cdots \ge P_{m-1} \ge P_m = P_{m+1} = \cdots = P_\infty$ to be $I_{m,f} = P_m/(1+a^2_{m,1}+\cdots+a^2_{m,f})$, $0 \le f \le m$, which reveals that the (m,f)th-order MMS interpolation error is smaller than the mth-order MMS prediction error as long as $f > 0$.

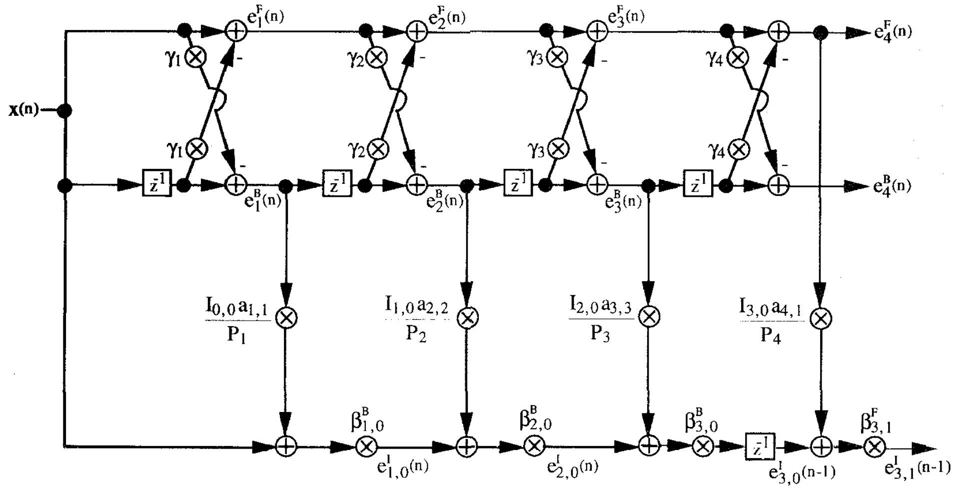

Fig. 2. Lattice structure realization of the (3, 1)st-order interpolation error filter.

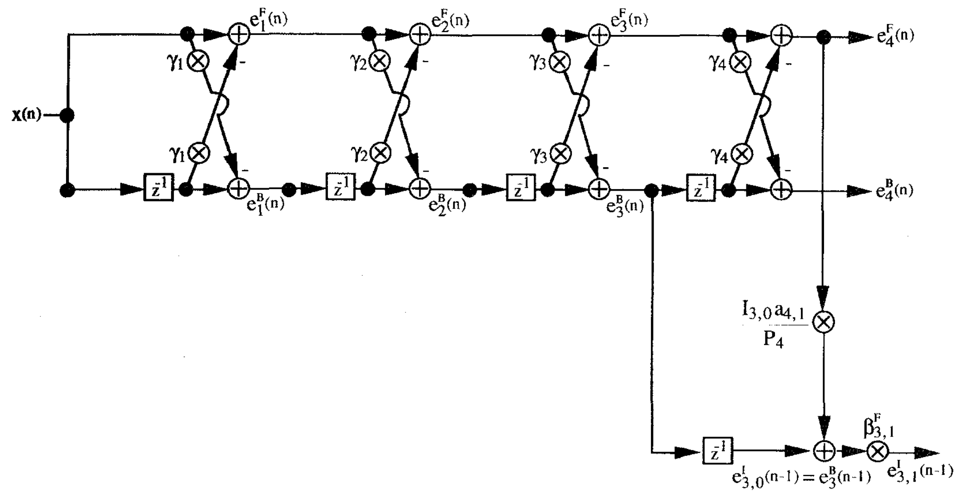

Fig. 3. Simplified realization of Fig. 2 by using $e_{1,0}^I(n) = e_1^B(n), e_{2,0}^I(n) = e_2^B(n)$, and $e_{3,0}^I(n) = e_3^B(n)$.

Moreover, the minimum interpolation error power

$$I_{p,p} = P_p \Big/ \left(1 + \sum_{i=1}^{p} a_{p,i}^2\right)$$

can be achieved when $m = f = p$.

III. LSL Interpolation Filters

In this section we develop the LS asymmetric interpolation lattice structure. Let $x(i)$ be the input signal to the asymmetric interpolation error filter of order (p, f), where $i = 1, 2, \cdots, n$ and n is the variable length of the input signal samples. The interpolation coefficient vector at time n is $\mathbf{b}_{p,f}^T(n-f) = [b_{(p,f),f}(n-f), \cdots, b_{(p,f),1}(n-f), 1, b_{(p,f),-1}(n-f), \cdots, b_{(p,f),-p}(n-f)]$ that will be optimized in the least-squares sense over the observation interval $1-f \leq i \leq n-f$ as follows: Let the $(q+1)$-by-1 input vector be given as $\mathbf{x}_{q+1}^T(i) = [x(i), x(i-1), \cdots, x(i-q)]$, $1-f \leq i \leq n-f$, where $q = p+f$. The (p, f)th order interpolation error at each time unit is $e_{p,f}^I(i) = \mathbf{b}_{p,f}^T(n-f)\mathbf{x}_{q+1}(i+f)$, $1-f \leq i \leq n-f$. Note that the interpolation coefficients in vector $\mathbf{b}_{p,f}(n-f)$ remain fixed during the observation interval $1-f \leq i \leq n-f$. Also note that the use of prewindowing is assumed, that is, $x(i) = 0$ for $i \leq 0$. The optimum interpolation coefficients can be determined by minimizing the sum of the (p, f)th-order interpolation error squares, $\sum_{i=1-f}^{n-f}(e_{p,f}^I(i))^2$, with respect to the interpolation coefficients in vector $\mathbf{b}_{p,f}(n-f)$. This operation will yield the following deterministic form of the augmented normal equation for the linear asymmetric interpolation

$$\mathbf{R}_{q+1}(n)\mathbf{b}_{p,f}(n-f) = \mathbf{i}_{p,f}(n-f) \tag{14}$$

in which $\mathbf{i}_{p,f}^T(n-f) = [\mathbf{O}_f^T \; I_{p,f}(n-f) \; \mathbf{O}_p^T]$ and $\mathbf{R}_{q+1}(n) = \mathbf{A}_{q+1}^T(n)\mathbf{A}_{q+1}(n)$ is the $(q+1)$-by-$(q+1)$ deterministic correlation matrix, where

$$\mathbf{A}_{q+1}^T(n) = \begin{bmatrix} x(1) & x(2) & & x(n) \\ 0 & x(1) & \cdots & x(n-1) \\ 0 & 0 & & x(n-2) \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & x(n-q) \end{bmatrix}.$$

TABLE I
COMPUTER SIMULATION RESULTS OF THE LS ASYMMETRIC INTERPOLATION LATTICE SOLUTION WITH THE AR(2) PROCESS INPUT

simulation	AR parameters and variance of the driving white-noise process			eigenvalue spread	steady-state values of the average mean-squared error $(n \to \infty)$			theoretical minimum mean-squared interpolation error
	a_1	a_2	σ_ε^2		$\langle(e_2^F(n))^2\rangle$	$\langle(e_4^F(n))^2\rangle$	$\langle(e_{2,2}^I(n-2))^2\rangle$	$E\{(e_{2,2}^I(n-2))^2\}_{min}^*$
#1	-.9750	.95	.0731	3	.073797	.073747	.025946	$\frac{.0731}{1+a_1^2+a_2^2} = .0256$
#2	-1.9114	.95	.0038	100	.005854	.005739	.00112	$\frac{.0038}{1+a_1^2+a_2^2} = .000684$

* $E\{(e_{2,2}^I(n-2))^2\}_{min}$ is the theoretical minimum mean-squared interpolation error which can be computed by using equation (6) or (8).

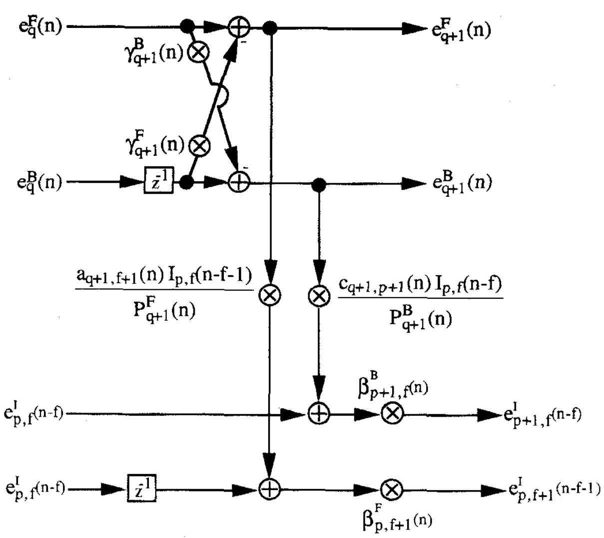

Fig. 4. Single-stage least-squares lattice realization of the $(p, f+1)$st-order and the $(p+1, f)$th-order interpolation error filter.

$\mathbf{O}_f$ and $\mathbf{O}_p$ in the vector $\mathbf{i}_{p,f}(n-f)$ are column vectors of f and p zeros, respectively, and $I_{p,f}(n-f)$ is the minimum value of the sum of the (p,f)th-order interpolation error square with $x(n)$ being the most recent signal sample used. When f and p are set to zero, respectively, in (14), the deterministic form of the augmented normal equation for the linear asymmetric interpolation will reduce to the following widely known deterministic form of the augmented normal equations for forward and backward predictions:

$$\mathbf{R}_{q+1}(n)\mathbf{a}_q(n) = \left[P_q^F(n), \mathbf{O}_q^T\right]^T \tag{15}$$

and

$$\mathbf{R}_{q+1}(n)\mathbf{c}_q(n) = \left[\mathbf{O}_q^T, P_q^B(n)\right]^T \tag{16}$$

where $\mathbf{a}_q^T(n) = [1, a_{q,1}(n), a_{q,2}(n), \cdots, a_{q,q}(n)]$ and $\mathbf{c}_q^T(n) = [c_{q,q}(n), \cdots, c_{q,2}(n), c_{q,1}(n), 1]$ are qth-order forward and backward prediction coefficients, respectively, and $P_q^F(n)$ and $P_q^B(n)$ are the minimum values of the sum of the qth-order forward and backward prediction-error squares , respectively. It is well known that the efficient order-update recursions used to solve the forward and backward prediction coefficients in (15) and (16) known as the LSL algorithm were first developed in [6]. The LS asymmetric interpolation lattice can be developed using the results of the LSL algorithm. The development begins with the realization that the $(p, f+1)$th-order augmented deterministic interpolation normal equation can be deduced from (14) as

$$\mathbf{R}_{q+2}(n)\mathbf{b}_{p,f+1}(n-f-1) = \left[\mathbf{0}_{f+1}^T, I_{p,f+1}(n-f-1, \mathbf{0}_p^T\right]^T \tag{17}$$

where

$$\begin{aligned}&\mathbf{b}_{p,f+1}^T(n-f-1)+\\ &[b_{(p,f+1),f+1}(n-f-1), \cdots,\\ &b_{(p,f+1),1}(n-f-1), 1, b_{(p,f+1),-1}(n-f-1), \cdots,\\ &b_{(p,f+1),-p}(n-f-1)],\ I_{p,f+1}(n-f-1)\end{aligned}$$

is the minimum value of the sum of the $(p, f+1)$th-order interpolation-error squares with $x(n)$ being the most recent signal sample used. The $(q+2)$-by-$(q+2)$ deterministic correlation matrix $\mathbf{R}_{q+2}(n)$ can be expressed as $\mathbf{R}_{q+2}(n) = \mathbf{A}_{q+2}^T(n)\mathbf{A}_{q+2}(n)$, where $\mathbf{A}_{q+2}^T(n)$ is defined to be

$$\mathbf{A}_{q+2}^T(n) = \begin{bmatrix} \mathbf{A}_{q+1}^T(n) \\ 0 \cdots 0\, x(1) \cdots x(n-q-1) \end{bmatrix}.$$

To obtain $\mathbf{b}_{p,f+1}(n-f-1)$, we invert the matrix $\mathbf{R}_{q+2}(n)$ in (17) by using the formula [5, p. 577] as follows:

$$\mathbf{R}_{q+2}^{-1}(n) = \begin{bmatrix} 0 & 0_{q+1}^T \\ 0_{q+1} & \mathbf{R}_{q+1}^{-1}(n-1) \end{bmatrix} + \frac{\mathbf{a}_{q+1}(n)\mathbf{a}_{q+1}^T(n)}{P_{q+1}^F(n)}.$$

As a result, the order-update recursion

$$\begin{aligned}\mathbf{b}_{p,f+1}(n-f-1) = \beta_{p,f+1}^F(n)\Bigg\{&\begin{bmatrix} 0 \\ \mathbf{b}_{p,f}(n-f-1) \end{bmatrix} \\ &+ \left(\frac{I_{p,f}(n-f-1)a_{q+1,f+1}(n)}{P_{q+1}^F(n)}\right)\mathbf{a}_{q+1}(n)\Bigg\}\end{aligned} \tag{18}$$

can be obtained by relating $I_{p,f}(n-f-1)$ and $I_{p,f+1}(n-f-1)$ as

$$I_{p,f+1}(n-f-1) = \beta_{p,f+1}^F(n)I_{p,f}(n-f-1) \tag{19}$$

where $\beta_{p,f+1}^F(n)$ can be found from the $(f+2)$nd row of (18) to be

$$\beta_{p,f+1}^F(n) = 1\Big/\left(1 + \frac{I_{p,f}(n-f-1)a_{q+1,f+1}^2(n)}{P_{q+1}^F(n)}\right). \tag{20}$$

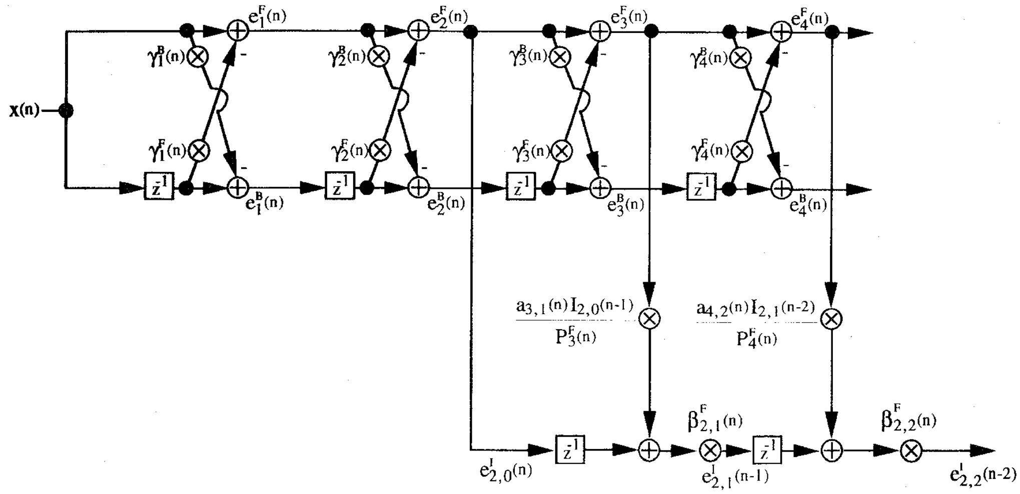

Fig. 5. Least-squares lattice implementation of the (2, 2)th-order interpolation error filter.

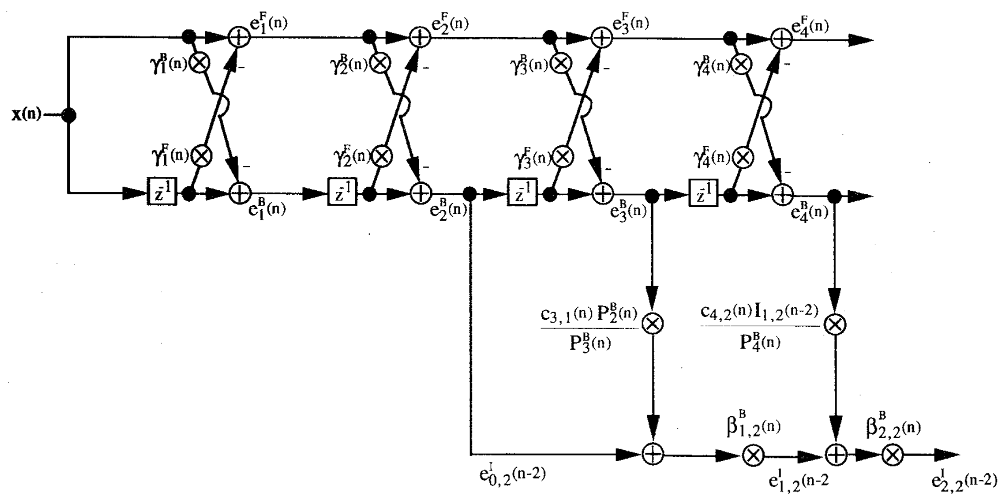

Fig. 6. Least-squares lattice implementation of the (2, 2)th-order interpolation error filter.

The $(p, f+1)$th-order LS asymmetric interpolation lattice structure can then be obtained by premultiplying both sides of (18) by row vector $[x(n), x(n-1), \cdots, x(n-q-1)]$. This yields

$$e_{p,f+1}^I(n-f-1) = \beta_{p,f+1}^F(n)\left\{e_{p,f}^I(n-f-1) + \left(\frac{I_{p,f}(n-f-1)a_{q+1,f+1}(n)}{P_{q+1}^F(n)}\right)e_{q+1}^F(n)\right\}. \tag{21}$$

The quantity $e_{p,f+1}^I(n-f-1)$ represents the interpolation error when one estimates the current signal sample, $x(n-f-1)$, from its p past and $(f+1)$ future neighboring samples by minimizing the sum of the $(p, f+1)$st-order interpolation error squares with the most recent signal sample used up to $x(n)$.

The order-update recursion for interpolation error when one additional past signal sample is used to estimate the current signal sample can be similarly obtained as follows:

$$e_{p+1,f}^I(n-f) = \beta_{p+1,f}^B(n)\left\{e_{p,f}^I(n-f) + \left(\frac{I_{p,f}(n-f)c_{q+1,p+1}(n)}{P_{q+1}^B(n)}\right)e_{q+1}^B(n)\right\} \tag{22}$$

where

$$\beta_{p+1,f}^B(n) = 1\Big/\left(1 + \frac{I_{p,f}(n-f)c_{q+1,p+1}^2(n)}{P_{q+1}^B(n)}\right) \tag{23}$$

and

$$I_{p+1,f}(n-f) = \beta_{p+1,f}^B(n)I_{p,f}(n-f) \tag{24}$$

is the minimum value of the sum of the $(p+1, f)$th-order interpolation-error squares with $x(n)$ being the most recent signal sample used. Equations (19)–(23) and (24) make up the LS asymmetric interpolation lattice solution. A single-stage LSL

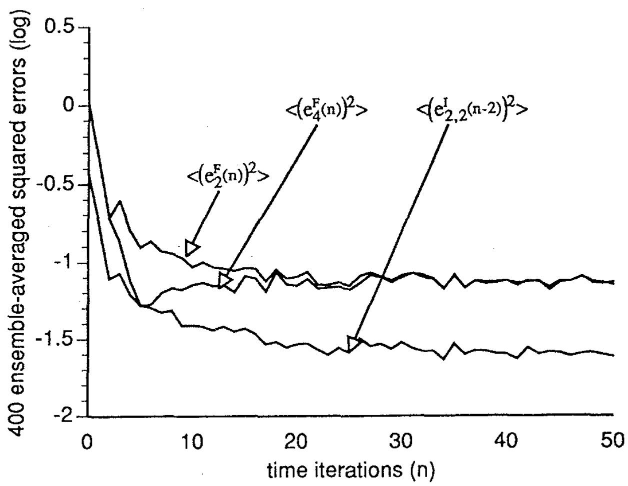

Fig. 7. Learning curves of the LSL algorithm for linear predictors and interpolator with eigenvalue spread of the AR(2) process being set to three.

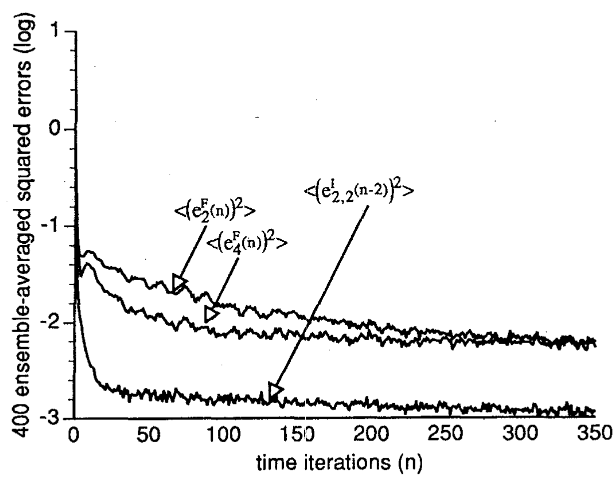

Fig. 8. Learning curves of the LSL algorithm for linear predictors and interpolator with eigenvalue spread of the AR(2) process being set to 100.

realization of $(p, f+1)$st-order and $(p+1, f)$th-order interpolation error filters using (21) and (22) is shown in Fig. 4. As an example, Figs. 5 and 6 show the two lattice implementations of the (2, 2)th-order interpolation error filter.

IV. Simulation Results

In this section, we describe a computer simulation experiment that compares the performances of the prediction lattice and interpolation lattice using the result of the LS interpolation lattice developed in Section III. For purposes of comparison, we use an AR(2) data process defined as $x(n)+a_1x(n-1)+a_2x(n-2)=\varepsilon(n)$ where the driving process, $\varepsilon(n)$, is a computer-generated sequence simulating a zero-mean Gaussian white noise process with variance σ_ε^2. The AR parameters a_1 and a_2 are listed in Table I and are chosen so that the AR process $x(n)$ has unity variance. The symbol $\langle\rangle$ is used to denote a 400-sample ensemble average. For convenience, the AR parameter values are from [5, p. 286]. The results of the simulations are presented in Figs. 7 and 8, corresponding to an eigenvalue spread of three and 100, respectively, using a $\log_{10}$ scale. In Fig. 7, the ensemble-averaged squared errors for $e_2^F(n)$ and $e_4^F(n)$ for $0 \leq n \leq 50$ and $e_{2,2}^I(n-2)$ for $2 \leq n \leq 52$ were computed over 400 trials each. In Fig. 8, the ensemble-averaged squared errors for $e_2^F(n), e_4^F(n)$ for $0 \leq n \leq 350$ and $e_{2,2}^I(n-2)$ for $2 \leq n \leq 352$ were computed, again with 400 trials. Each trial used an independent realization of the white noise process $\varepsilon(n)$. As is clearly seen from Figs. 7 and 8, interpolation yields smaller steady-state values of the average squared error than prediction does. Also, the learning curve of the interpolation lattice shown in Fig. 8 appears to converge to the steady-state value faster than its prediction counterparts for high eigenvalue spreads. These results reveal that the LSL interpolation, which makes better use of the correlation between the nearest neighboring samples than the LSL prediction, may achieve much better performance than that of the LSL prediction.

V. Conclusion

Although the symmetric interpolation is most often used, however, the asymmetric interpolation lattice developed in this paper provides a computationally efficient and structurally flexible implementation for the solution of the interpolation lattice problems. Furthermore, it forms a bridge between the widely known forward prediction, backward prediction, and symmetric interpolation, which can all be viewed as special cases of asymmetric interpolation. As a result, the asymmetric interpolation lattice provides a broader interpretation and a more thorough understanding of the linear prediction and linear interpolation theories.

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