

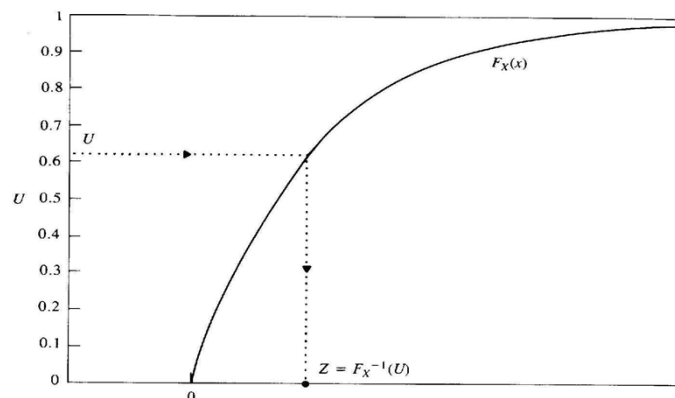
Computer methods for generating random variables

機率實習 (一)

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Transformation Method

- Generate U uniformly distributed in $[0,1]$
- Let $Z = F_X^{-1}(U)$

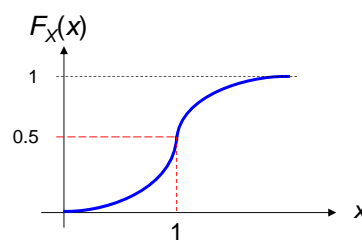
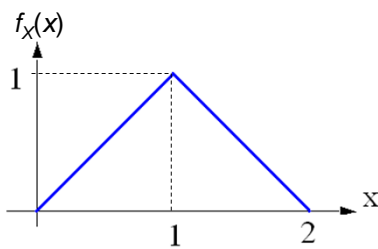


Example

- 使用Transformation Method 產生一個pdf為三角分佈的 random number

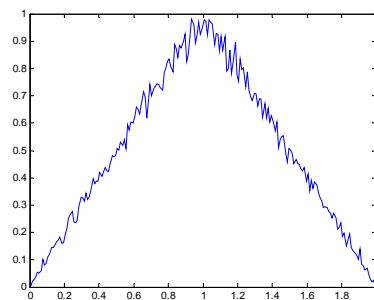
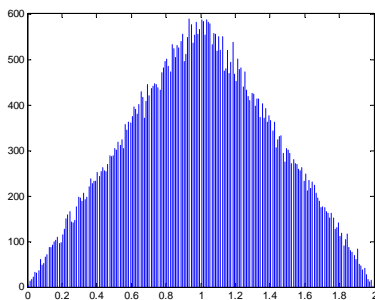
(a) $0 \leq x < 1 \rightarrow F_x(x) = \frac{x^2}{2}$ $0 \leq u < 0.5 \rightarrow u = \frac{x^2}{2} \Rightarrow x = \sqrt{2u},$

(b) $1 \leq x \leq 2 \rightarrow F_x(x) = 1 - \frac{(2-x)^2}{2}$ $0.5 \leq u \leq 1 \rightarrow u = 1 - \frac{(2-x)^2}{2}$
 $\Rightarrow x = 2 - \sqrt{2-2u}$



Simulation Results

- Sample點為60000點，分200堆做統計
- 將其除以60000後所得到的值為，範圍在1~1.01內的CDF為**0.00978333**，
 - 1~1.01內平均的PDF為**0.032675/0.01=0.978333**。
 - Mean=0.999805, Variance = 0.167199



MATLAB Example(Uniform)

- `N=input('please input number of samples? ');`
- `x=rand(1,N);`
- `figure(1)`
- `subplot(2,2,1:2);`
- `plot(x);`
- `subplot(2,2,3);`
- `hist(x,40);`
- `hx=hist(x,40);`

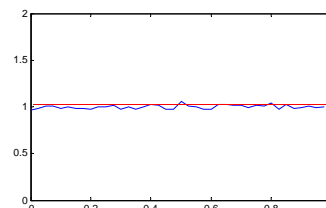
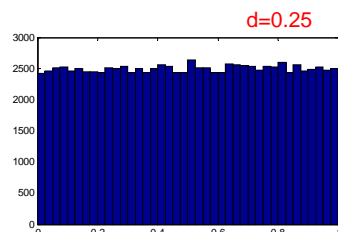
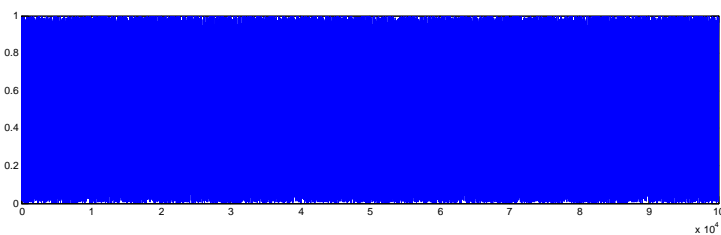
- `d=1/40;`
- `hx=hist(x,40);`
- `hy=hx/N;`
- `py=hy/d;`

- `i=0:39;`
- `px=i*d;`
- `subplot(2,2,4)`
- `plot(px,py);`

- `'average =',mean(x)`
- `pause;`

Results (Uniform)

- 100000 points, 40 groups, [0,1]

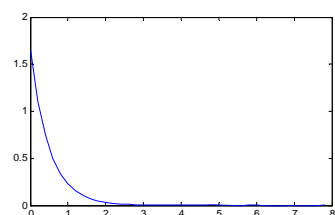
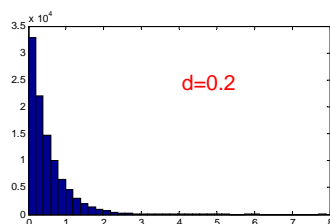
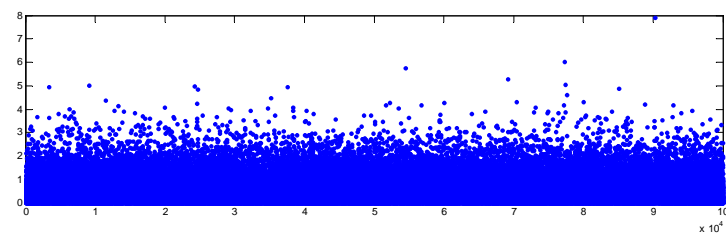


MATLAB Example(Exp.)

- numda=input('please input numda? ');
- y=-1/numda*log(x);
- figure (2)
- subplot(2,2,1:2);
- plot(y);
- subplot(2,2,3);
- hist(y,40);
- d=input('input space = ');
- hx=hist(y,40);
- hy=hx/N;
- py=hy/d;
- i=0:39;
- px=i*d;
- subplot(2,2,4)
- plot(px,py);
- 'average =',mean(y)
- pause;

Results(Exponential R.V.)

- 100000 points, 40 groups ($\lambda = 2$)



Central-limit

```
t=input('please input number of R.V.? ');
figure (3)
y=0;
for i=1:t
    x=rand(1,N);
    x=-1/numda*log(x);
    x=x+y;
    y=x;
    pause(1);

    subplot(2,2,1:2);
    plot(x);

    subplot(2,2,3);
    hist(x,40);

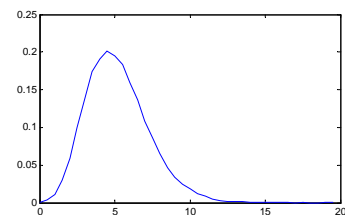
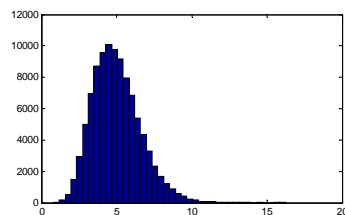
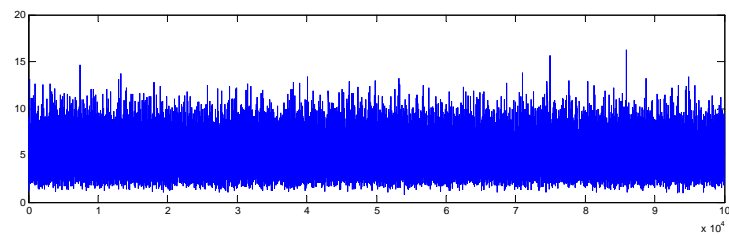
    'average =', mean(x)
end

d=input('input space = ');
hx=hist(x,40);
hy=hx/N;
py=hy/d;

i=0:39;
px=i*d;
subplot(2,2,4)
plot(px,py);
```

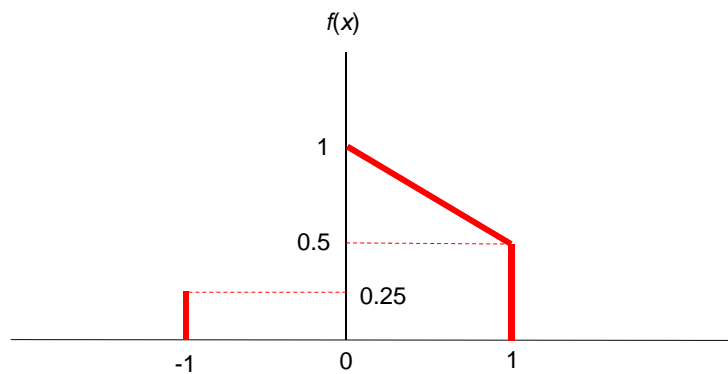
Central-limit

- Exponential ($\lambda = 2$), $X_1+X_2+\dots+X_{10}$



Assignment 1

- Generate X with pdf $f(x)$



Assignment 2

- Generate X with pdf $f(x)$
- Generate $Y = X^2$ and verify by finding $f(y)$
- Generate $Y = X_1 + X_2$ and verify by finding $f(y)$
– $X_1 + X_2$ with the same pdf $f(x)$

