Supplementary Information For "An Indirect-Reciprocity Game Theoretic Framework for Device-to-Device Multicast"

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I. Proof of Theorem 1

Theorem 1: There exists a unique stationary reputation distribution of the whole population for any given optimal action rule, and the stationary reputation distribution is $\boldsymbol{P_t}$'s eigenvector with the corresponding eigenvalue one.

Proof: Let $f(\lambda)$ denote the characteristic polynomial of matrix $\boldsymbol{P_t}$, which is given by

$$f(\lambda) = |\lambda \boldsymbol{I} - \boldsymbol{P_t}| = \begin{vmatrix} \lambda - P_{t_{0,0}} & -P_{t_{0,1}} & \cdots & -P_{t_{0,L-1}} \\ -P_{t_{0,0}} & \lambda - P_{t_{1,1}} & \cdots & -P_{t_{1,L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ -P_{t_{L-1,0}} & -P_{t_{L-1,1}} & \cdots & \lambda - P_{t_{L-1,L-1}} \end{vmatrix}. \tag{1}$$

With the property of determinant, if the columns from the second to the last of $\lambda \boldsymbol{I} - \boldsymbol{P_t}$ are added to the first column, the determinant is unchanged. Besides, the sum of each row in $\boldsymbol{P_t}$ is equal to 1, i.e., $\forall i \in \{0, 1, 2, ..., L-1\}$, there is

$$\sum_{j=0}^{L-1} P_{t_{i,j}} = 1. \tag{2}$$

Therefore, the characteristic polynomial of matrix $\boldsymbol{P_t}$ is rewritten as

$$f(\lambda) = |\lambda \boldsymbol{I} - \boldsymbol{P_t}| = \begin{vmatrix} \lambda - 1 & P_{t_{0,1}} & \cdots & P_{t_{0,L-1}} \\ \lambda - 1 & \lambda - P_{t_{1,1}} & \cdots & P_{t_{1,L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda - 1 & P_{t_{L-1,1}} & \cdots & \lambda - P_{t_{L-1,L-1}} \end{vmatrix}. \tag{3}$$

Noticing that the elements in the first column of (3) are all $\lambda - 1$, we can conclude that $\lambda = 1$ is a root of the characteristic equation, i.e., 1 is an eigenvalue of $\boldsymbol{P_t}$. Hence, $\boldsymbol{P_t}$ and its transpose must have an eigenvalue of 1, and the eigenvector corresponding to this eigenvalue is the solution to (12) in [1]. ■

II. Proof of Theorem 2

Theorem 2: The optimal action in (17) of [1] is an ESS if the cost-to-gain ratio c_e/g satisfies $0 < c_e/g < \frac{(1-\lambda)\beta W G_{\min}}{\bar{\eta} P_{\max}}$, where $\bar{\eta} = \sum_{l=1}^{L} l\eta_l$, $W = w_b \bar{B}\bar{N} + w_e \bar{P}$, $G_{\min} = \min_{\rho'} \sum\limits_{m} (G_{L,m} - G_{f(\rho'),m})$, and $\rho' = \rho(l')$.

Proof: Given the SL s and the LDM where only the outermost l ($l \in \mathcal{L}$) rings are not distributed with RNs, $\forall i, j$, the optimal action should be $a^*_{i,j} = P_l$. With P_l, the HN can serve all the RNs. According to the social norm in (3) of [1], the HN is assigned an immediat reputation $\Omega_{s,L} = L$.

Then let $\bar{\eta} = \sum_{l=1}^{L} l\eta_l$ and $W = w_b \bar{B}\bar{N} + w_e \bar{P}$, we have

$$\hat{r}(a^*_{i,j}) = e_{\Omega_{s,L}} = e_L, \tag{4}$$

$$\boldsymbol{r}(a^*_{i,j}) = (1-\lambda) \sum_{l} e_{\Omega_{s,l}} G_{L,l} + \lambda e_i = (1-\lambda) G_{L,:} + \lambda e_i, \tag{5}$$

$$t(a^*_{i,j}) = \frac{\sum_{l=1}^{L} l[(1-\lambda)G_{L,l} + \lambda e_i]}{\bar{\eta}}, \tag{6}$$

and

$$\begin{aligned} & F_i(a^*, a^*) \quad (7) \\ &= -s d_{BH} - c_e P_l + g\eta W \frac{\sum_{l=1}^{L} l[(1-\lambda)G_{L,l} + \lambda e_i]}{\bar{\eta}} + \delta \sum_k \sum_m r_k(a_{i,m}) U_{k,m} q_m \\ &= -s d_{BH} - c_e P_l + g\eta W \frac{\sum_{l=1}^{L} l[(1-\lambda)G_{L,l} + \lambda e_i]}{\bar{\eta}} + \delta\lambda U_{i,:} \boldsymbol{q}^T + \delta(1-\lambda) \sum_m G_{L,m} U_{m,:} \boldsymbol{q}^T, \end{aligned}$$

where $\boldsymbol{q} = [q_1, ..., q_L]$ and $\boldsymbol{q}^T$ is the transpose of $\boldsymbol{q}$.

$\forall a_{i,j} \neq a^*_{i,j}$, if $a_{i,j} = P'_l$ and $P'_l > P_l$, we have $\rho(l') = \rho(l)$, and thus $\hat{r}(a_{i,j}) = \hat{r}(a^*_{i,j})$, $\boldsymbol{r}(a_{i,j}) = \boldsymbol{r}(a^*_{i,j})$ and $t(a_{i,j}) = t(a^*_{i,j})$. Since $c_e > 0$ and $c_e P'_l > c_e P_l$ is always hold, we have $F_i(a^*, a^*) > F_i(a^*, a^*)$.

If $a_{i,j} = P_l'$ and $P_l' < P_l$, denoting $\rho(l') = \rho'$, we have

$$\hat{r}(a_{i,j}) = e_{\Omega_{s,f(\rho')}} = e_{f(\rho')}, \tag{8}$$

$$\boldsymbol{r}(a_{i,j}) = (1-\lambda)\sum_{l} e_{\Omega_{s,l}} G_{f(\rho'),l} + \lambda e_i = (1-\lambda) G_{f(\rho'),:} + \lambda e_i, \tag{9}$$

$$t(a_{i,j}) = \frac{\sum_{l=1}^{L} l[(1-\lambda)G_{f(\rho'),l} + \lambda e_i]}{\bar{\eta}}, \tag{10}$$

and

$$\begin{aligned} & F_i(a_{i,j}, a^*) \\ &= -sd_{BH} - c_e P_{l'} + g\eta W \tfrac{\sum_{l=1}^{L} l[(1-\lambda)G_{f(\rho'),l}+\lambda e_i]}{\bar{\eta}} + \delta \sum_k \sum_m r_k(a_{i,m}) U_{k,m} q_m \\ &= -sd_{BH} - c_e P_{l'} + g\eta W \tfrac{\sum_{l=1}^{L} l[(1-\lambda)G_{f(\rho'),l}+\lambda e_i]}{\bar{\eta}} + \delta\lambda U_{i,:}\boldsymbol{q}^T + \delta(1-\lambda)\sum_m G_{f(\rho'),m} U_{m,:}\boldsymbol{q}^T. \end{aligned} \tag{11}$$

Combining (7) and (11), we have

$$\begin{aligned} & F_i(a^*, a^*) - F_i(a_{i,j}, a^*) \\ &= -c_e(P_l - P_{l'}) + g\eta W \tfrac{\sum_{l=1}^{L} l(1-\lambda)[G_{L,l} - G_{f(\rho'),l}]}{\bar{\eta}} + \delta(1-\lambda)\sum_m (G_{L,m} - G_{f(\rho'),m}) U_{m,:}\boldsymbol{q}^T \\ &> -c_e P_{\max} + g\eta W \tfrac{\sum_{l=1}^{L} l(1-\lambda)G_{\min}}{\bar{\eta}} \end{aligned} \tag{12}$$

where $G_{\min} = \min_{\rho'} \sum_m (G_{L,m} - G_{f(\rho'),m})$.

Then for $F_i(a^*, a^*) > F_i(a^*, a^*)$ to be hold, we should have

$$g/c_e > \frac{P_{\max}\bar{\eta}}{(1-\lambda)WG_{\min}}. \tag{13}$$

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References

[1] Y. Chen, B. Zhang, M. Wang, T. Xu, and Z. Han,"An Indirect-Reciprocity Game Theoretic Framework for Device-to-Device Multicast", submitted to ICC 2019.