Supplementary Information for “Long-Term Contract Design for Traffic Off-Loading in Heterogeneous Cloud Radio Access Networks”

Min Wang[12], Biling Zhang[12], Jung-Lang Yu[3], Zhu Han[45]
[1]School of Network Education,
Beijing University of Posts and Telecommunications, Beijing, 100876, China
[2]The State Key Laboratory of Integrated Services Networks,
Xidian University, P. R. China
Email: {wang_min, bilingzhang}@bupt.edu.cn
[3]Department of Electrical Engineering,
Fu Jen Catholic University, New Taipei City, 24205, Taiwan
Email: yujl@ee.fju.edu.tw
[4]Department of Electrical and Computer Engineering,
University of Houston, Houston, TX 77004, USA
[5]Department of Computer Science and Engineering,
Kyung Hee University, Seoul, South Korea
Email: zhan2@uh.edu

CNR Partitioning: The channel-to-noise ratio (CNR) is defined as

$$\gamma = \frac{|h|^2}{PL(d)N0B}, \tag{1}$$

where h stands for the Rayleigh fading power gain and follows a unit-mean exponential distribution, $N0$ is the noise power spectral density, B is the channel bandwidth, and $PL(d) = 10^{(30+20log_{10}d)/10}$ is the path loss between the offloaded UE and the RRH with the distance of d.

Since γ is a monotonic function of d, if we quantize the distance interval $[0, d_{max}]$, where d_{max} is the radius of the RRH, to M levels $\mathcal{D} = \{D_1, D_2 \ldots, D_M\}$ with $D_1 = d_{max}$ and $D_M = 0$, then $\forall D_i \in \mathcal{D}$, the corresponding CNR can be obtained as

$$\Gamma_i = \frac{|h|^2}{PL(D_i)N0B}. \tag{2}$$

For $d \geq d_{max}$, we quantize it to the $0th$ level and set $\Gamma_0 = 0$. Moreover, we set the $|h|^2 = E[|h|^2] = 1$. In such a way, we successfully obtain the $M + 1$ CNR states.

Lemma 1: Given a type-$\boldsymbol{\theta}_k$ RRH, for any state $\mathbf{s}_k^{l,m}, \mathbf{s}_k^{i,j}$, we have $R_k^{l,m} > R_k^{i,j}$ if and only if $T_k^{l,m} > T_k^{i,j}$.

Proof: First,we prove the sufficiency: if $T_k^{l,m} > T_k^{i,j}$, then $R_k^{l,m} > R_k^{i,j}$. According to the IC constraints for type-$\boldsymbol{\theta}_k$ with state $\mathbf{s}_k^{i,j}$, we have

$$T_k^{i,j} - c\frac{2^{\frac{R_k^{i,j} N_i}{B}} - 1}{\Gamma_j} \geq T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m} N_i}{B}} - 1}{\Gamma_j}, \tag{3}$$

which can be transformed to be

$$c\frac{2^{\frac{R_k^{l,m} N_i}{B}} - 2^{\frac{R_k^{i,j} N_i}{B}}}{\Gamma_j} \geq T_k^{l,m} - T_k^{i,j}. \tag{4}$$

Since $T_k^{l,m} > T_k^{i,j}$, then $R_k^{l,m} > R_k^{i,j}$.

Next, we prove the necessity: if $R_k^{l,m} > R_k^{i,j}$, then $T_k^{l,m} > T_k^{i,j}$. Similar to the first case, we start with the IC constraints for type-$\boldsymbol{\theta}_k$ with state $\mathbf{s}_k^{l,m}$, and we can obtain

$$T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m} N_l}{B}} - 1}{\Gamma_m} \geq T_k^{i,j} - c\frac{2^{\frac{R_k^{i,j} N_l}{B}} - 1}{\Gamma_m}, \tag{5}$$

which can be transformed to be

$$T_k^{l,m} - T_k^{i,j} \geq c\frac{2^{\frac{R_k^{l,m} N_l}{B}} - 2^{\frac{R_k^{i,j} N_l}{B}}}{\Gamma_m}. \tag{6}$$

As $R_k^{l,m} > R_k^{i,j}$, then $T_k^{l,m} > T_k^{i,j}$.

Lemma 1 indicates that the RRH offering more transmission data should be given with more rewards by the LPN, and vice versa. ■

Lemma 2: *(Monotonicity of the Transmission rate for different states)*Given a type-$\boldsymbol{\theta}_k$ RRH, if state $\mathbf{s}_k^{l,m} > \mathbf{s}_k^{i,j}$, then $R_k^{l,m} > R_k^{i,j}$.

Proof: Considering the IC constraints for type-$\boldsymbol{\theta}_k$ with state $\mathbf{s}_k^{l,m}$, we have

$$T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m} N_l}{B}} - 1}{\Gamma_m} \geq T_k^{i,j} - c\frac{2^{\frac{R_k^{i,j} N_l}{B}} - 1}{\Gamma_m}. \tag{7}$$

Due to the IC constraints for type-$\boldsymbol{\theta}_k$ with state $\mathbf{s}_k^{i,j}$, we have

$$T_k^{i,j} - c\frac{2^{\frac{R_k^{i,j} N_i}{B}} - 1}{\Gamma_j} \geq T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m} N_i}{B}} - 1}{\Gamma_j}. \tag{8}$$

By combining (7) and (8), we can obtain

$$\Gamma_m(2^{\frac{R_k^{l,m}N_i}{B}} - 2^{\frac{R_k^{i,j}N_i}{B}}) - \Gamma_j(2^{\frac{R_k^{l,m}N_l}{B}} - 2^{\frac{R_k^{i,j}N_l}{B}}) \geq 0. \tag{9}$$

Since $N_l < N_i$, $\Gamma_m > \Gamma_j$, by reductio, then $R_k^{l,m} > R_k^{i,j}$.

Lemma 2 indicates that the RRH at a higher state should be given with more rewards. ■

Lemma 3: Given a type-$\boldsymbol{\theta}_k$ RRH, if state $\mathbf{s}_k^{l,m} > \mathbf{s}_k^{i,j}$, then $U_k^{l,m} > U_k^{i,j}$.

Proof: According to the IC constraints for type-$\boldsymbol{\theta}_k$ with state $\mathbf{s}_k^{l,m}$, we have

$$T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m}N_l}{B}} - 1}{\Gamma_m} \geq T_k^{i,j} - c\frac{2^{\frac{R_k^{i,j}N_l}{B}} - 1}{\Gamma_m}. \tag{10}$$

Then, we can derive

$$\begin{aligned} T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m}N_l}{B}} - 1}{\Gamma_m} \geq & T_k^{i,j} - c\frac{2^{\frac{R_k^{i,j}N_i}{B}} - 1}{\Gamma_j} \\ & + \frac{c}{\Gamma_m\Gamma_j}\{(\Gamma_m - \Gamma_j)(2^{\frac{R_k^{i,j}N_i}{B}} - 1) + \Gamma_j(2^{\frac{R_k^{i,j}N_i}{B}} - 2^{\frac{R_k^{i,j}N_l}{B}})\}. \end{aligned} \tag{11}$$

Since $N_l < N_i$, $\Gamma_m > \Gamma_j$, then $T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m}N_l}{B}}-1}{\Gamma_m} > T_k^{i,j} - c\frac{2^{\frac{R_k^{i,j}N_i}{B}}-1}{\Gamma_j}$. Meanwhile, the type-$\boldsymbol{\theta}_k$ RRH at the higher state $\mathbf{s}_k^{l,m}$ has larger transition probabilities to other high states, compared with the state $\mathbf{s}_k^{i,j}$. Hence, $\delta\sum_{l'=1}^{L}\sum_{m'=1}^{M} P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_k^{l',m'} > \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M} P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{i,j})U_k^{l',m'}$, which can also be proved by simulation. Combining the above two equalities, we have

$$\begin{aligned} T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m}N_l}{B}} - 1}{\Gamma_m} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M} P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_k^{l',m'} & \\ > T_k^{i,j} - c\frac{2^{\frac{R_k^{i,j}N_i}{B}} - 1}{\Gamma_j} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M} P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{i,j})U_k^{l',m'}, & \end{aligned} \tag{12}$$

i.e., $U_k^{l,m} > U_k^{i,j}$.

Lemma 3 indicates that the RRH at a higher state will obtain more long-term utility. ■

Lemma 4: Given a state (N_l, Γ_m), if $\boldsymbol{\theta}_k > \boldsymbol{\theta}_{k'}$, then $U_k^{l,m} > U_{k'}^{l,m}$.

Proof: Considering the IC constraints for the state (N_l, Γ_m) with different long-term types in (16), we have

$$T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m} N_l}{B}} - 1}{\Gamma_m} + \delta \sum_{l'=1}^{L} \sum_{m'=1}^{M} P_k(\mathbf{s}_k^{l',m'} | \mathbf{s}_k^{l,m}) U_k^{l',m'} \\ \geq T_{k'}^{l,m} - c\frac{2^{\frac{R_{k'}^{l,m} N_l}{B}} - 1}{\Gamma_m} + \delta \sum_{l'=1}^{L} \sum_{m'=1}^{M} P_k(\mathbf{s}_k^{l',m'} | \mathbf{s}_k^{l,m}) U_{k'}^{l',m'}, \tag{13}$$

Moreover, for the same state (N_l, Γ_m), the type-$\boldsymbol{\theta}_k$ RRH has larger transition probabilities to high states and has smaller transition probabilities to low states, compared with the type-$\boldsymbol{\theta}_{k'}$ RRH. Hence, $\delta \sum\limits_{l'=1}^{L} \sum\limits_{m'=1}^{M} P_k(\mathbf{s}_k^{l',m'} | \mathbf{s}_k^{l,m}) U_{k'}^{l',m'} > \delta \sum\limits_{l'=1}^{L} \sum\limits_{m'=1}^{M} P_{k'}(\mathbf{s}_{k'}^{l',m'} | \mathbf{s}_{k'}^{l,m}) U_{k'}^{l',m'}$, which can also be proved by simulation. Therefore, we can derive

$$T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m} N_l}{B}} - 1}{\Gamma_m} + \delta \sum_{l'=1}^{L} \sum_{m'=1}^{M} P_k(\mathbf{s}_k^{l',m'} | \mathbf{s}_k^{l,m}) U_k^{l',m'} \\ > T_{k'}^{l,m} - c\frac{2^{\frac{R_{k'}^{l,m} N_l}{B}} - 1}{\Gamma_m} + \delta \sum_{l'=1}^{L} \sum_{m'=1}^{M} P_{k'}(\mathbf{s}_{k'}^{l',m'} | \mathbf{s}_{k'}^{l,m}) U_{k'}^{l',m'}, \tag{14}$$

i.e., $U_k^{l,m} > U_{k'}^{l,m}$.

Lemma 4 indicates that for the same state, RRHs of a higher long-term type will achieve more utility from the long-term perspective. ■

Lemma 5 *(IRL:Individual Rational Constraint for the lowest type with the lowest state):* If only the $\boldsymbol{\theta}_1$ RRH with state $\mathbf{s}_k^{1,1}$ among all IR constraints binds, then the other IR constraints will automatically hold, i.e., IR constraints can be replaced by

$$U_1^{1,1} = T_1^{1,1} - c\frac{2^{\frac{R_1^{1,1} N_1}{B}} - 1}{\Gamma_1} + \delta \sum_{l'=1}^{L} \sum_{m'=1}^{M} P_1(\mathbf{s}_1^{l',m'} | \mathbf{s}_1^{1,1}) U_1^{l',m'} = 0. \tag{15}$$

Proof: Lemma 3 and Lemma 4 indicate that $U_k^{l,m} \geq U_1^{1,1}$, $\forall k \in \mathcal{K},\ l \in \mathcal{L},$ and $m \in \mathcal{M}$. Hence, $U_1^{1,1} = 0$ can guarantee that all other $U_k^{l,m} > 0$. ■

Lemma 6 *(LDICs:Local Downward Incentive Constraints):*

(I) *(LDICs for Instantaneous States)* Given a type-$\boldsymbol{\theta}_k$ RRH: if the LDICs are satisfied for all states

$\mathbf{s}_k^{l,m}$, $\forall l \in \mathcal{L}, m \in \mathcal{M}$, i.e.,

$$\begin{aligned} T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m}N_l}{B}}-1}{\Gamma_m} \geq \max\{T_k^{l-1,m-1} - c\frac{2^{\frac{R_k^{l-1,m-1}N_l}{B}}-1}{\Gamma_m}, \\ T_k^{l-1,m} - c\frac{2^{\frac{R_k^{l-1,m}N_l}{B}}-1}{\Gamma_m}, T_k^{l,m-1} - c\frac{2^{\frac{R_k^{l,m-1}N_l}{B}}-1}{\Gamma_m}\}, \end{aligned} \tag{16}$$

then IC constraints for a given type will hold for any $i \leq l, j \leq m$.

(II) *(LDICs for Long-term Types)* Given a state (N_l, Γ_m): if the LDICs are satisfied for all $\boldsymbol{\theta}_k$, $\forall k \in \mathcal{K}$, i.e.,

$$\begin{aligned} T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m}N_l}{B}}-1}{\Gamma_m} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M} P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_k^{l',m'} \\ \geq T_{k-1}^{l,m} - c\frac{2^{\frac{R_{k-1}^{l,m}N_l}{B}}-1}{\Gamma_m} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M} P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_{k-1}^{l',m'}, \end{aligned} \tag{17}$$

then IC constraints for a given state will hold for any $k' \leq k$.

(III) *(LDICs for mixed conditions)* For different types RRHs at different states: if the LDICs in (16) and (17) are satisfied, then IC constraints will hold for any $k' \leq k$ with any $i \leq l, j \leq m$.

Proof: Firstly, we prove the LDICs for instantaneous states in (I). Secondly, we prove the LDICs for log-term types in (II). Finally, we prove the LDICs for mixed conditions in (III).

(I) Given a type-$\boldsymbol{\theta}_k$ RRH, considering the IC constraints for different states, we have

$$T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m}N_l}{B}}-1}{\Gamma_m} \geq T_k^{l-1,m-1} - c\frac{2^{\frac{R_k^{l-1,m-1}N_l}{B}}-1}{\Gamma_m}, \tag{18}$$

and

$$T_k^{l-1,m-1} - c\frac{2^{\frac{R_k^{l-1,m-1}N_{l-1}}{B}}-1}{\Gamma_{m-1}} \geq T_k^{l-2,m-2} - c\frac{2^{\frac{R_k^{l-2,m-2}N_{l-1}}{B}}-1}{\Gamma_{m-1}}. \tag{19}$$

According to Lemma 1, since $\mathbf{s}_k^{l-1,m-1} > \mathbf{s}_k^{l-2,m-2}$, ie., $R_k^{l-1,m-1} > R_k^{l-2,m-2}$. By substituting it into (19), we have

$$T_k^{l-1,m-1} - c\frac{2^{\frac{R_k^{l-1,m-1}N_l}{B}}-1}{\Gamma_m} \geq T_k^{l-2,m-2} - c\frac{2^{\frac{R_k^{l-2,m-2}N_l}{B}}-1}{\Gamma_m}. \tag{20}$$

Combining (18) and (20), we can derive

$$T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m}N_l}{B}} - 1}{\Gamma_m} \geq T_k^{l-2,m-2} - c\frac{2^{\frac{R_k^{l-2,m-2}N_l}{B}} - 1}{\Gamma_m}. \tag{21}$$

Similar to the derivation process of (21), we can obtain the following two inequalities

$$T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m}N_l}{B}} - 1}{\Gamma_m} \geq T_k^{l-2,m} - c\frac{2^{\frac{R_k^{l-2,m}N_l}{B}} - 1}{\Gamma_m}, \tag{22}$$

$$T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m}N_l}{B}} - 1}{\Gamma_m} \geq T_k^{l,m-2} - c\frac{2^{\frac{R_k^{l,m-2}N_l}{B}} - 1}{\Gamma_m} \tag{23}$$

Then, by combining (21) (22) and (23), we can rewrite the above three inequalities as

$$\begin{aligned} T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m}N_l}{B}} - 1}{\Gamma_m} \geq &\max\{T_k^{l-2,m-2} - c\frac{2^{\frac{R_k^{l-2,m-2}N_l}{B}} - 1}{\Gamma_m}, \\ &T_k^{l-2,m} - c\frac{2^{\frac{R_k^{l-2,m}N_l}{B}} - 1}{\Gamma_m}, T_k^{l,m-2} - c\frac{2^{\frac{R_k^{l,m-2}N_l}{B}} - 1}{\Gamma_m}\}. \end{aligned} \tag{24}$$

(II) Given a state (N_l, Γ_m), considering the IC constraints for different types, we have

$$\begin{aligned} &T_k^{l,m} - c\frac{2^{\frac{R_k^{l,m}N_l}{B}} - 1}{\Gamma_m} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M} P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_k^{l',m'} \\ &\geq T_{k-1}^{l,m} - c\frac{2^{\frac{R_{k-1}^{l,m}N_l}{B}} - 1}{\Gamma_m} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M} P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_{k-1}^{l',m'}. \end{aligned} \tag{25}$$

and

$$\begin{aligned} &T_{k-1}^{l,m} - c\frac{2^{\frac{R_{k-1}^{l,m}N_l}{B}} - 1}{\Gamma_m} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M} P_{k-1}(\mathbf{s}_{k-1}^{l',m'}|\mathbf{s}_{k-1}^{l,m})U_{k-1}^{l',m'} \\ &\geq T_{k-2}^{l,m} - c\frac{2^{\frac{R_{k-2}^{l,m}N_l}{B}} - 1}{\Gamma_m} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M} P_{k-1}(\mathbf{s}_{k-1}^{l',m'}|\mathbf{s}_{k-1}^{l,m})U_{k-2}^{l',m'}. \end{aligned} \tag{26}$$

Due to Lemma 4, $\boldsymbol{\theta}_{k-1} > \boldsymbol{\theta}_{k-2}$, i.e., $U_{k-1}^{l',m'} > U_{k-2}^{l',m'}$. Transforming (26), we have

$$\begin{aligned} &T_{k-1}^{l,m} - c\frac{2^{\frac{R_{k-1}^{l,m}N_l}{B}}-1}{\Gamma_m} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M}P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_{k-1}^{l',m'} \\ &\geq T_{k-2}^{l,m} - c\frac{2^{\frac{R_{k-2}^{l,m}N_l}{B}}-1}{\Gamma_m} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M}P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_{k-2}^{l',m'}. \end{aligned} \tag{27}$$

Combining (25) and (27), we can derive

$$\begin{aligned} &T_{k}^{l,m} - c\frac{2^{\frac{R_{k}^{l,m}N_l}{B}}-1}{\Gamma_m} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M}P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_{k}^{l',m'} \\ &\geq T_{k-2}^{l,m} - c\frac{2^{\frac{R_{k-2}^{l,m}N_l}{B}}-1}{\Gamma_m} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M}P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_{k-2}^{l',m'}. \end{aligned} \tag{28}$$

(III) Considering the IC constraints for deferent types and states, combing (16) and (17), we have

$$\begin{aligned} &T_{k}^{l,m} - c\frac{2^{\frac{R_{k}^{l,m}N_l}{B}}-1}{\Gamma_m} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M}P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_{k}^{l',m'} \\ &\qquad \geq \max\{T_{k-1}^{l-1,m-1} - c\frac{2^{\frac{R_{k-1}^{l-1,m-1}N_l}{B}}-1}{\Gamma_m}, T_{k-1}^{l-1,m} - c\frac{2^{\frac{R_{k-1}^{l-1,m}N_l}{B}}-1}{\Gamma_m}, \\ &\qquad T_{k-1}^{l,m-1} - c\frac{2^{\frac{R_{k-1}^{l,m-1}N_l}{B}}-1}{\Gamma_m}\} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M}P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_{k-1}^{l',m'}. \end{aligned} \tag{29}$$

Combing (24) with $k = k-2$ and (28), we can get

$$\begin{aligned} &T_{k}^{l,m} - c\frac{2^{\frac{R_{k}^{l,m}N_l}{B}}-1}{\Gamma_m} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M}P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_{k}^{l',m'} \\ &\qquad \geq \max\{T_{k-2}^{l-2,m-2} - c\frac{2^{\frac{R_{k-2}^{l-2,m-2}N_l}{B}}-1}{\Gamma_m}, T_{k-2}^{l-2,m} - c\frac{2^{\frac{R_{k-2}^{l-2,m}N_l}{B}}-1}{\Gamma_m}, \\ &\qquad T_{k-2}^{l,m-2} - c\frac{2^{\frac{R_{k-2}^{l,m-2}N_l}{B}}-1}{\Gamma_m}\} + \delta\sum_{l'=1}^{L}\sum_{m'=1}^{M}P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_{k-2}^{l',m'}. \end{aligned} \tag{30}$$

■

Based on the above lemmas, the LPN's long-term utility maximization problem can be further repre-

$$\max_{(R_k^{i,j},T_k^{i,j})} U_{LPN}$$
$$(i)\ T_1^{1,1}-c\frac{2^{\frac{R_1^{1,1}N_1}{B}}-1}{\Gamma_1}+\delta\sum_{l'=1}^{L}\sum_{m'=1}^{M}P_1(\mathbf{s}_1^{l',m'}|\mathbf{s}_1^{1,1})U_1^{l',m'}=0,$$
$$(ii)\ \forall\ k\in\mathcal{K},\ l\in\mathcal{L},\ \text{and}\ m\in\mathcal{M},$$
$$T_k^{l,m}-c\frac{2^{\frac{R_k^{l,m}N_l}{B}}-1}{\Gamma_m}=\max\{T_k^{l-1,m-1}-c\frac{2^{\frac{R_k^{l-1,m-1}N_l}{B}}-1}{\Gamma_m},\ T_k^{l-1,m}-c\frac{2^{\frac{R_k^{l-1,m}N_l}{B}}-1}{\Gamma_m},\ T_k^{l,m-1}-c\frac{2^{\frac{R_k^{l,m-1}N_l}{B}}-1}{\Gamma_m}\},$$
$$(iii)\ \text{Given a state}\ (N_l,\Gamma_m),\ \forall k\in\mathcal{K},$$
$$T_k^{l,m}-c\frac{2^{\frac{R_k^{l,m}N_l}{B}}-1}{\Gamma_m}+\delta\sum_{l'=1}^{L}\sum_{m'=1}^{M}P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_k^{l',m'}=T_{k-1}^{l,m}-c\frac{2^{\frac{R_{k-1}^{l,m}N_l}{B}}-1}{\Gamma_m}+\delta\sum_{l'=1}^{L}\sum_{m'=1}^{M}P_k(\mathbf{s}_k^{l',m'}|\mathbf{s}_k^{l,m})U_{k-1}^{l',m'},$$
$$(iv)\ \forall\ l\in\mathcal{L},\ \text{and}\ m\in\mathcal{M},\ U_1^{l,m}<U_2^{l,m}<\cdots<U_K^{l,m},$$
$$(v)\ \forall\ k\in\mathcal{K},\ l\in\mathcal{L},\ \text{and}\ \ m\in\mathcal{M},\ R_k^{1,m}<R_k^{2,m}<\cdots<R_k^{L,m},\ \text{and}\ R_k^{l,1}<R_k^{l,2}<\cdots<R_k^{l,M}. \tag{31}$$

sented by (31). By simplifying the equality constraints in (31), we can conclude

$$T_1^{1,1}=c\frac{2^{\frac{R_1^{1,1}N_1}{B}}-1}{\Gamma_1}-\delta\sum_{l'=1}^{L}\sum_{m'=1}^{M}P_1(\mathbf{s}_1^{l',m'}|\mathbf{s}_1^{1,1})U_1^{l',m'}, \tag{32}$$

$$\begin{aligned}T_1^{l,m}=c\frac{2^{\frac{R_k^{l,m}N_l}{B}}-1}{\Gamma_m}+max\{T_1^{l-1,m-1}-c\frac{2^{\frac{R_1^{l-1,m-1}N_l}{B}}-1}{\Gamma_m},T_1^{l-1,m}-c\frac{2^{\frac{R_1^{l-1,m}N_l}{B}}-1}{\Gamma_m},\\ T_1^{l,m-1}-c\frac{2^{\frac{R_1^{l,m-1}N_l}{B}}-1}{\Gamma_m}\},\end{aligned} \tag{33}$$

$$T_k^{l,m}=T_1^{l,m}+\sum_{i=1}^{k-1}\{c\frac{2^{\frac{R_{i+1}^{l,m}N_l}{B}}-2^{\frac{R_i^{l,m}N_l}{B}}}{\Gamma_m}-\delta\sum_{l'=1}^{L}\sum_{m'=1}^{M}P_{i+1}(\mathbf{s}_{i+1}^{l',m'}|\mathbf{s}_{i+1}^{l,m})(U_{i+1}^{l',m'}-U_i^{l',m'})\}. \tag{34}$$

Note the optimization function in (31) is a Bellman equation of $U_{LPN,k}^{l,m}$, and thus finding the optimal contract item $(R_k^{l,m*},T_k^{l,m*})$ is an MDP. Hence, we adopt a modified value iteration algorithm to solve the MDP and the details of the algorithm are listed in Algorithm 1.

Algorithm 1 : Find the Optimal Contract Using Value Iteration

1. Given the tolerance $\varepsilon = 0.01$ and set $\varepsilon_1 = 1$, and initialize $\mathbf{R}^*$ with $\mathbf{R}^0$, which satisfies (v) in (31).
2. While $\varepsilon_1 > \varepsilon$
   - Set $\varepsilon_2 = 1$
   - Set $T_1^{1,1} = 0$, and initialize $U_{LPN,k}^{l,m} = 0$, $\forall k, \forall l, \forall m$.
   - While $\varepsilon_2 > \varepsilon$ or (iv) and (v) in (31) are not satisfied
     - Compute reward T_k^{l,m^*} using (33) and (34).
     - Obtain $\hat{U}_{LPN,k}^{l,m} = (gR_k^{l,m} - T_k^{l,m}) + \delta \sum\limits_{l'=1}^{L} \sum\limits_{m'=1}^{M} P_k(\mathbf{s}_k^{l',m'} | \mathbf{s}_k^{l,m}) \hat{U}_{LPN,k}^{l',m'}$.
     - Find the optimal transmission rate $\hat{R}_k^{l,m^*} = \arg \underset{R_k^{l,m}}{max}\ \hat{U}_{LPN,k}^{l,m}$.
     - Update the parameter ε_2 by $\varepsilon_2 = ||\hat{U}_{LPN,k}^{l,m} - U_{LPN,k}^{l,m}||^2$.
     - Update $U_{LPN,k}^{l,m}$ with $U_{LPN,k}^{l,m} = \hat{U}_{LPN,k}^{l,m}$.
     - Compute utility $U_k^{l,m} = V_k^{l,m} + \delta \sum\limits_{l'=1}^{L} \sum\limits_{m'=1}^{M} P_k(\mathbf{s}_k^{l',m'} | \mathbf{s}_k^{l,m}) U_k^{l',m'}$.
     - End
   - Update the parameter ε_1 by $\varepsilon_1 = ||\hat{\mathbf{R}}^* - \mathbf{R}^*||^2$.
   - Update $\mathbf{R}^*$ with $\mathbf{R}^* = \hat{\mathbf{R}}^*$.

   End