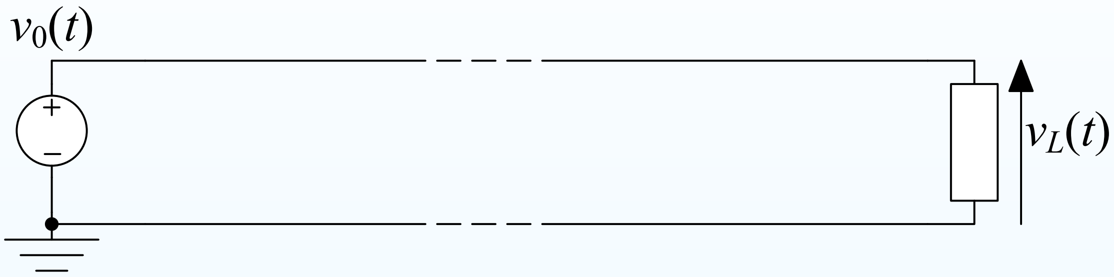
17: Transmission Lines

Transmission Lines



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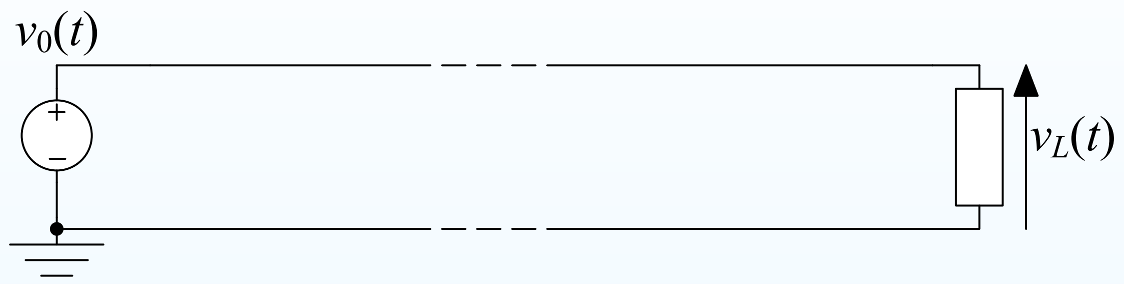

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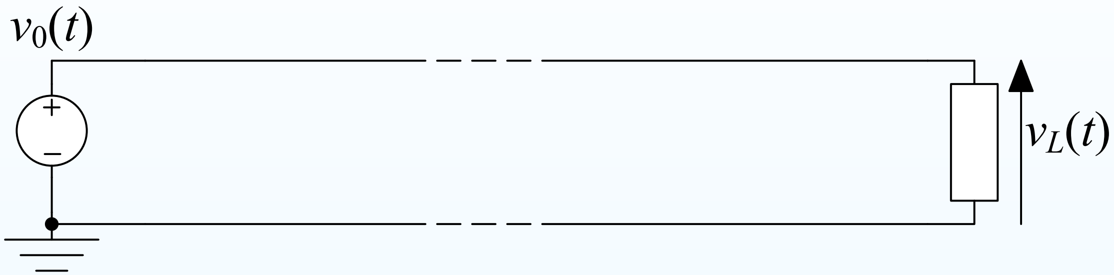

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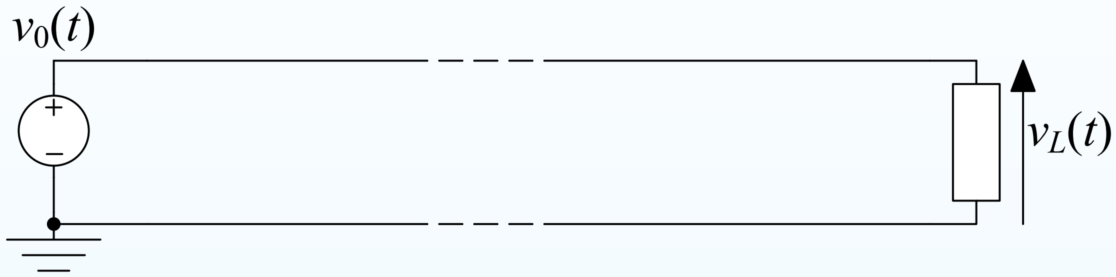

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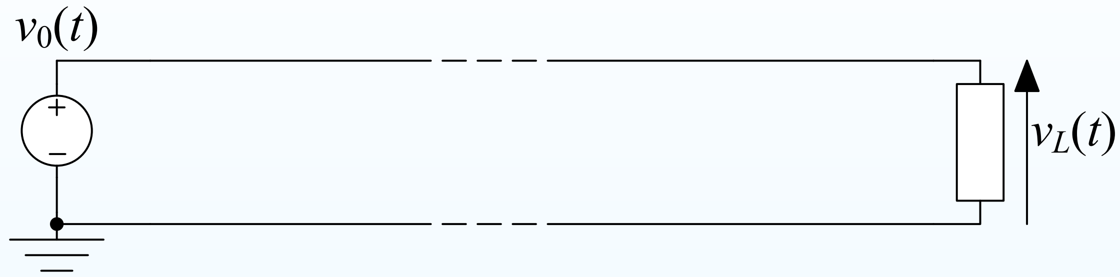

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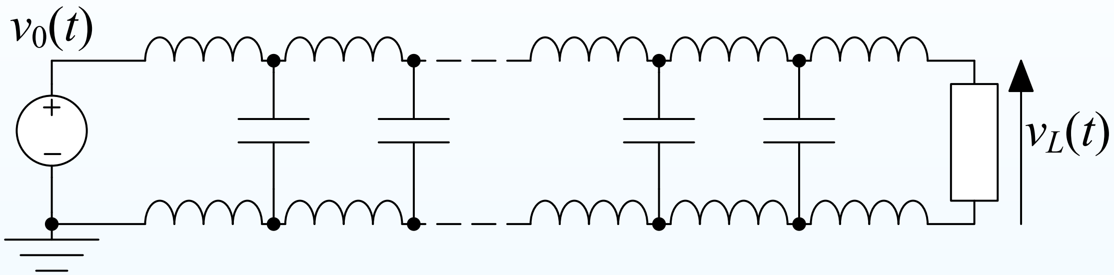

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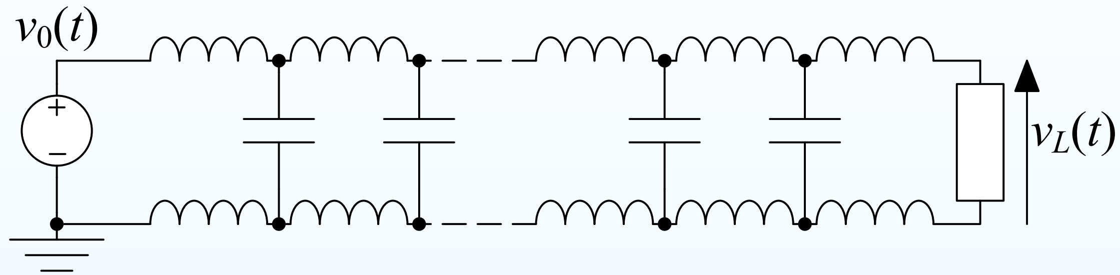

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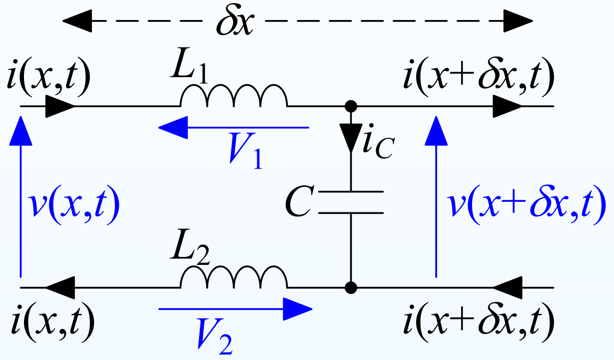
Transmission Line Equations

A short section of line δx long:

$v(x,t)$ and $i(x,t)$ depend on both position and time.



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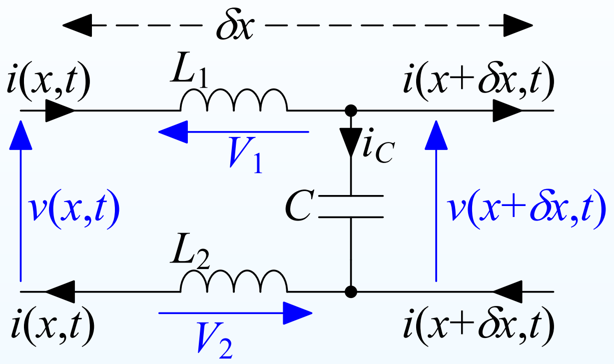

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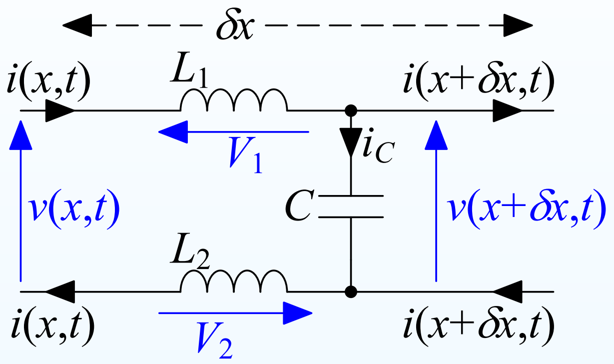

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Basic Equations

KVL: $v(x,t) = V_2 + v(x+\delta x,t) + V_1$

KCL: $i(x,t) = i_C + i(x+\delta x,t)$

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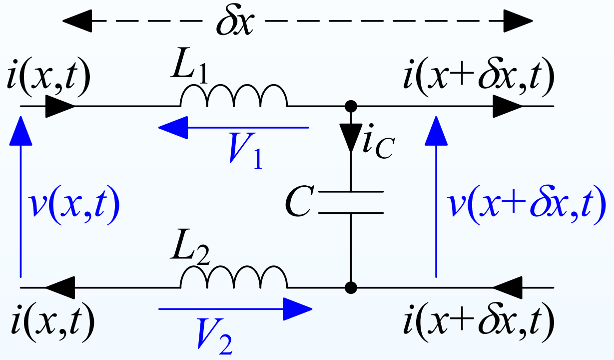

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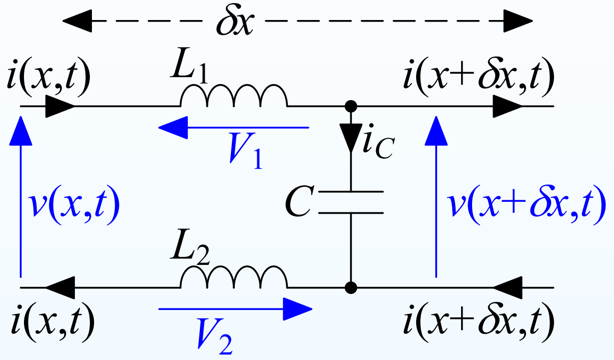

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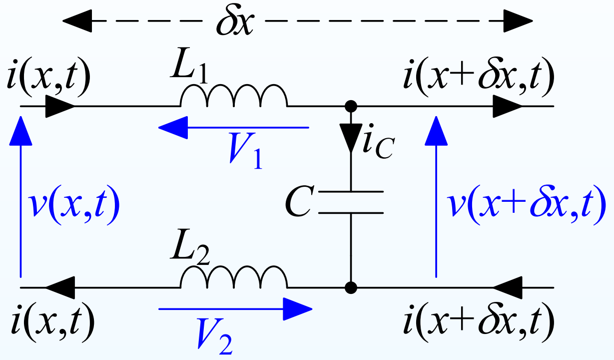

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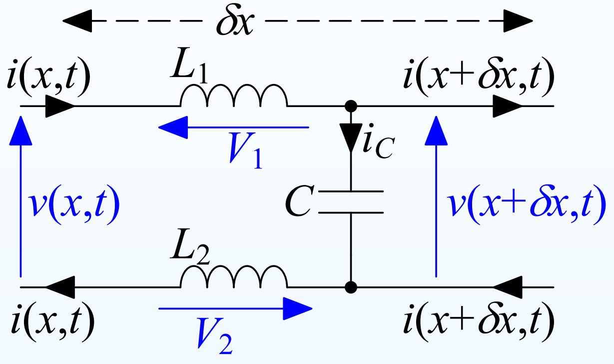

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$$C_0\frac{\partial v}{\partial t} = -\frac{\partial i}{\partial x}$$
$$L_0\frac{\partial i}{\partial t} = -\frac{\partial v}{\partial x}$$

where $C_0 = \frac{C}{\delta x}$ is the capacitance per unit length (Farads/m) and $L_0 = \frac{L_1+L_2}{\delta x}$ is the total inductance per unit length (Henries/m).

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General solution: $v(t,x) = f(t - \frac{x}{u}) + g(t + \frac{x}{u})$

$$i(t,x) = \frac{f(t-\frac{x}{u}) - g(t+\frac{x}{u})}{Z_0}$$

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Forward Wave

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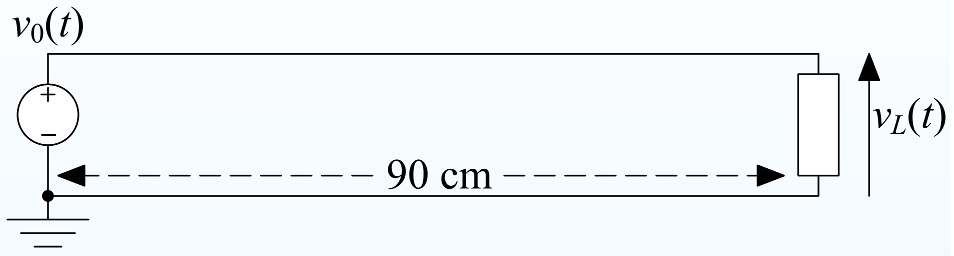
$u = 15$ cm/ns

and $g(t) \equiv 0$

$\Rightarrow v(x,t) = f\left(t - \frac{x}{u}\right)$



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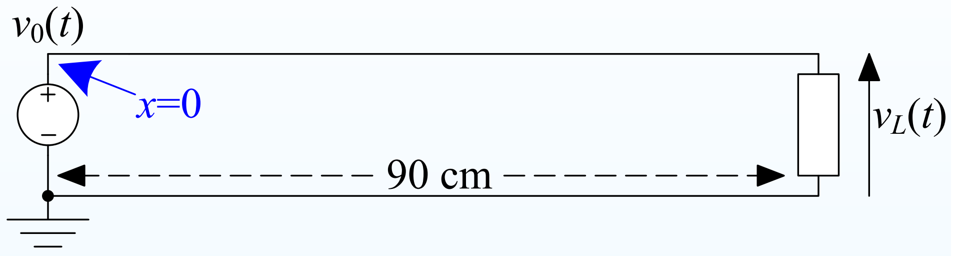

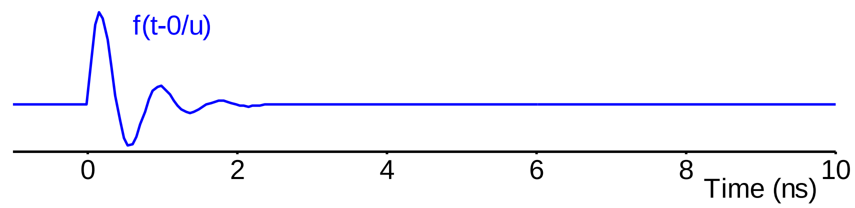

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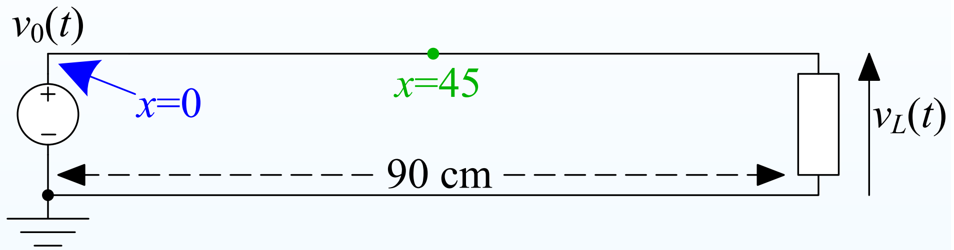

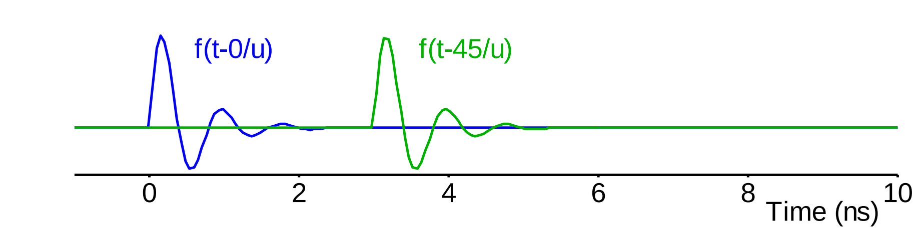

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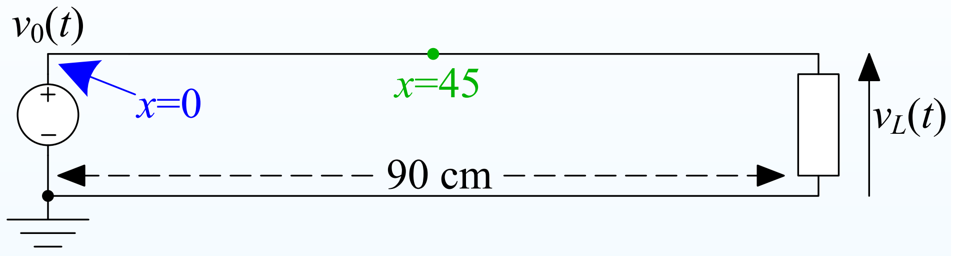

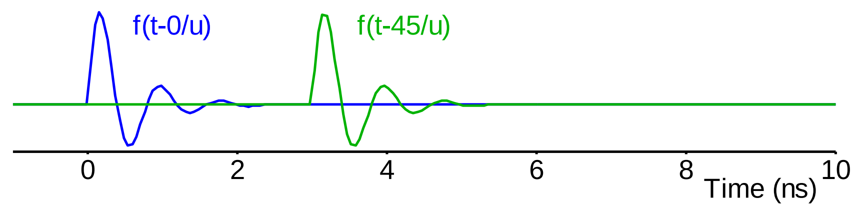

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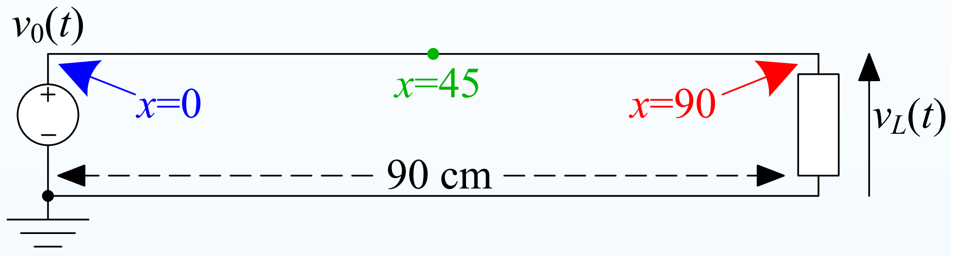

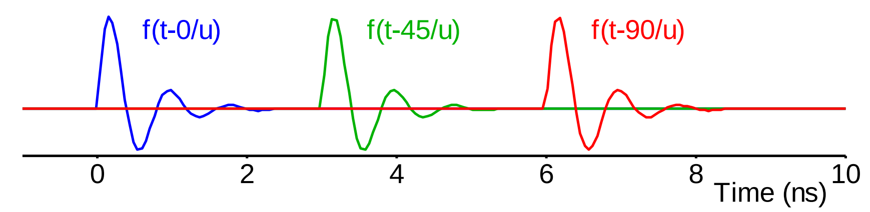

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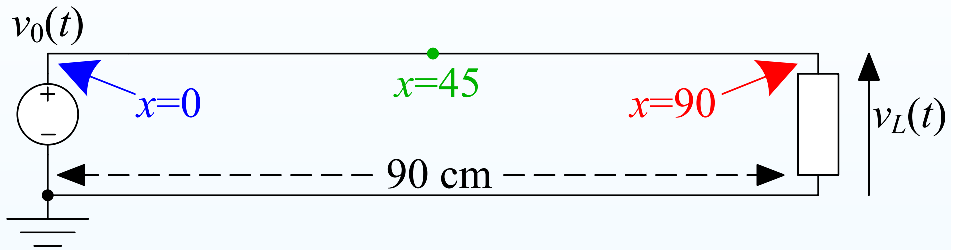

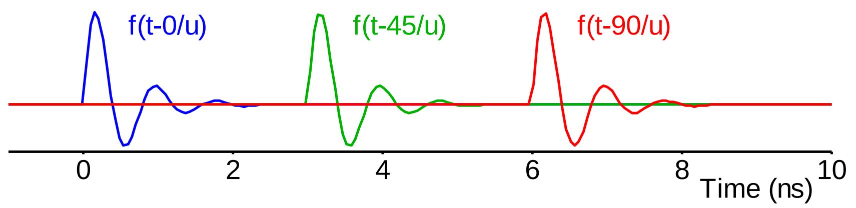

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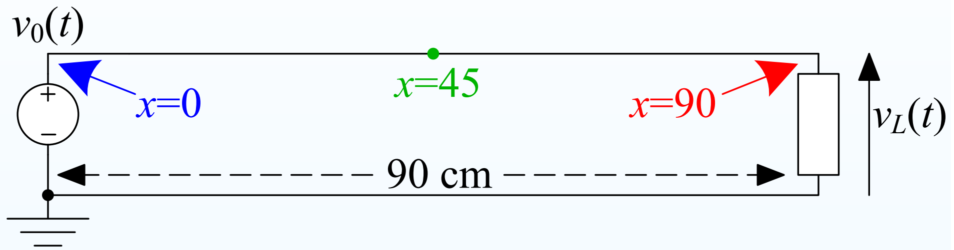

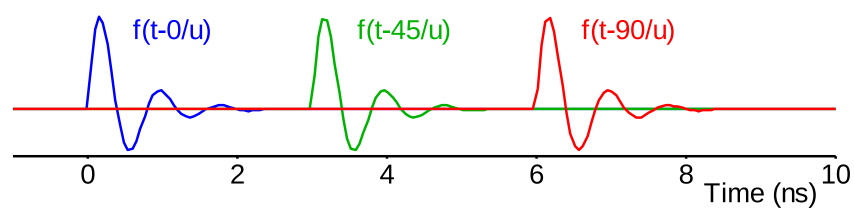

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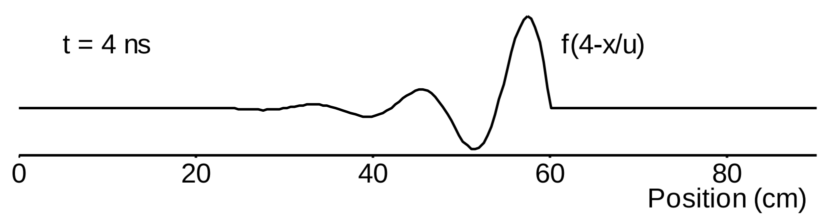
Snapshot at $t_0 = 4\,\text{ns}$:

the waveform has just arrived at the point $x = ut_0 = 60$ cm.



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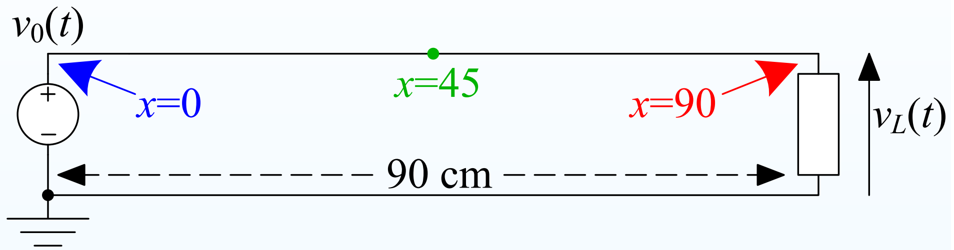

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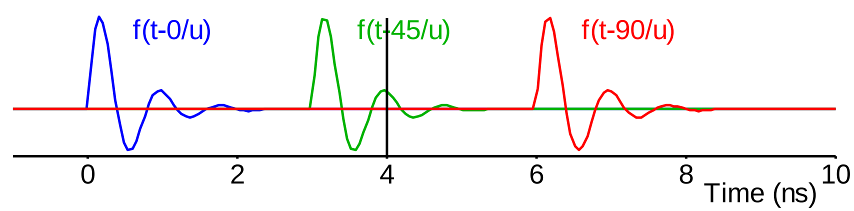

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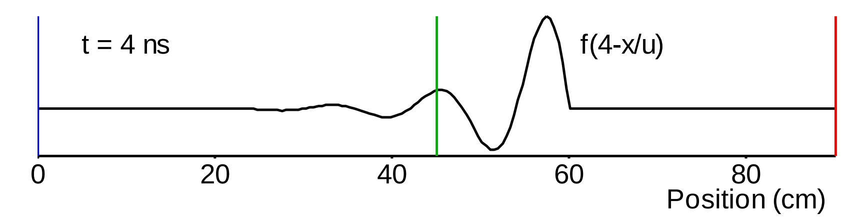

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and $g(t) \equiv 0$

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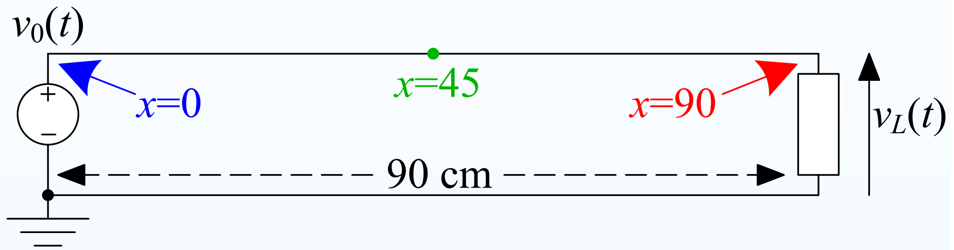
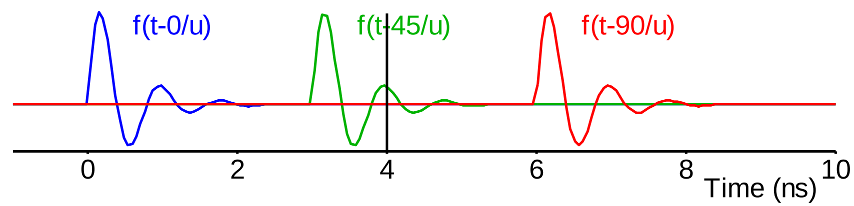


- At $x = 0$ cm [▲], $v_S(t) = f(t - \frac{0}{u})$



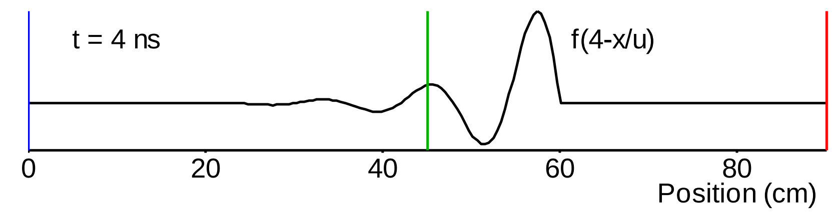
- At $x = 45$ cm [▲], $v(45,t) = f(t - \frac{45}{u})$

  $f(t - \frac{45}{u})$ is exactly the same as $f(t)$ but delayed by $\frac{45}{u} = 3$ ns.
- At $x = 90$ cm [▲], $v_R(t) = f(t - \frac{90}{u})$; now delayed by $6\,\text{ns}$.

Waveform at $x = 0$ completely determines the waveform everywhere else.

Snapshot at $t_0 = 4\,\text{ns}$:

the waveform has just arrived at the point $x = ut_0 = 60$ cm.



$f(t - \frac{x}{u})$ is a wave travelling forward (i.e. towards $+$x) along the line.

Forward + Backward Waves

Similarly $g(t + \frac{x}{u})$ is a wave travelling backwards, i.e. in the $-x$ direction.

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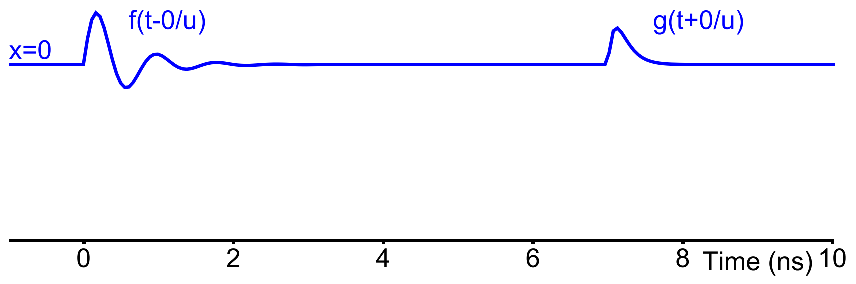

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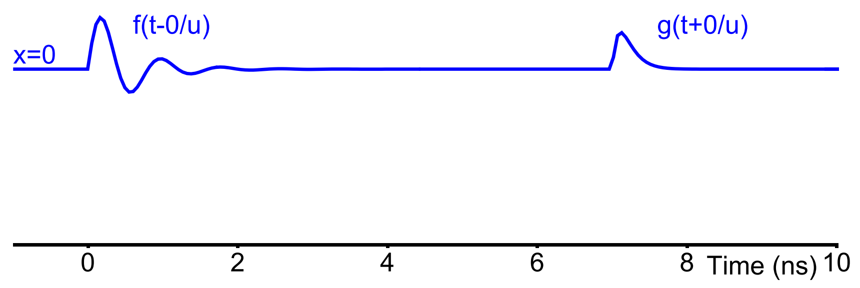

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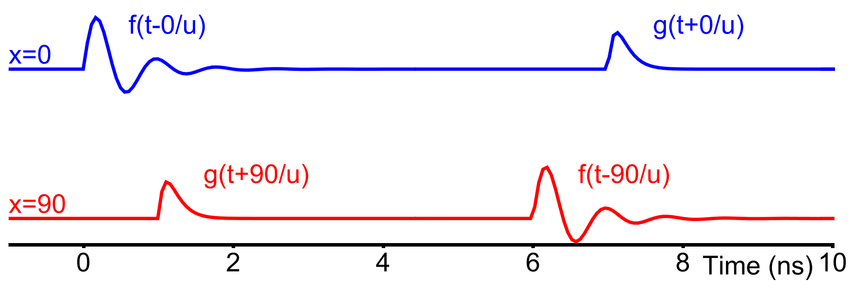

At $x = 0$ cm [▲],
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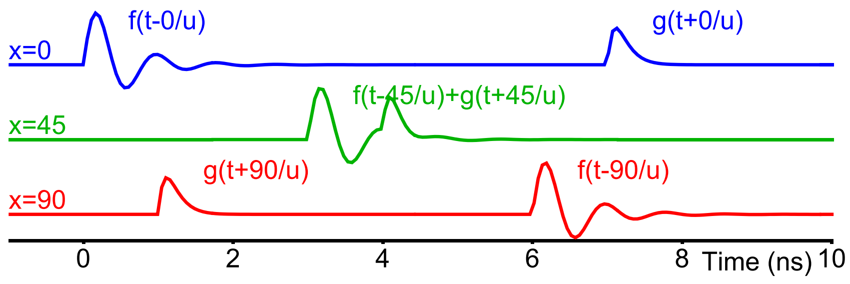

At $x = 45$ cm [▲], g is only 1 ns behind f and they add together.
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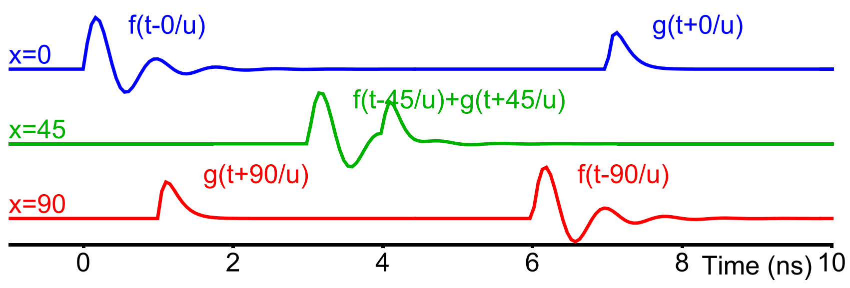

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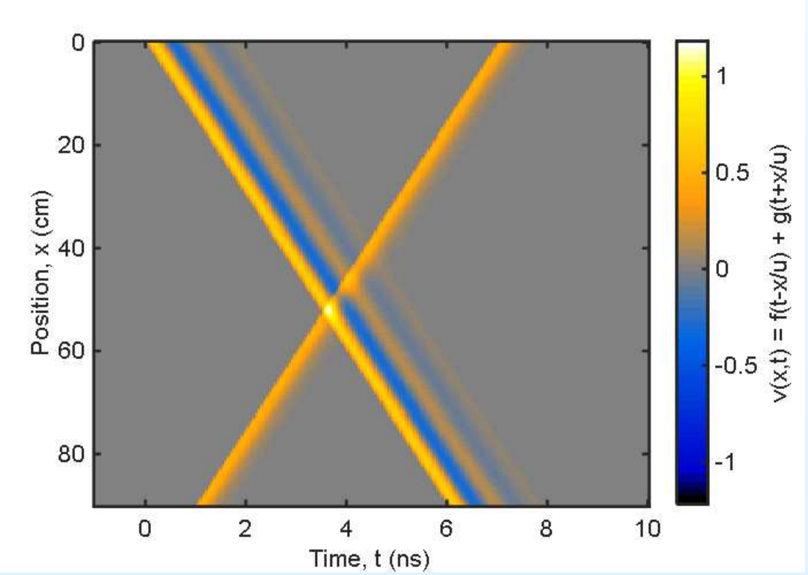
A vertical line on the diagram gives a snapshot of the entire line at a time instant t.



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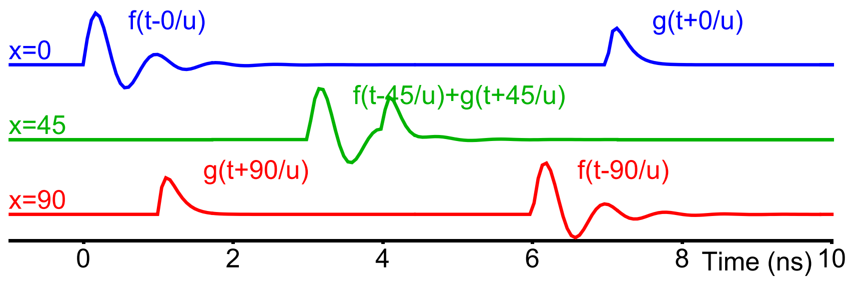

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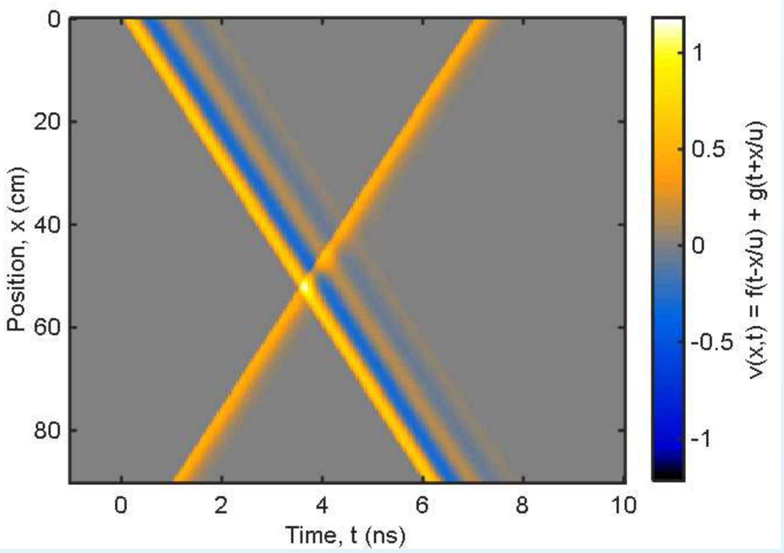
f and g first meet at $t = 3.5$ and $x = 52.5$.



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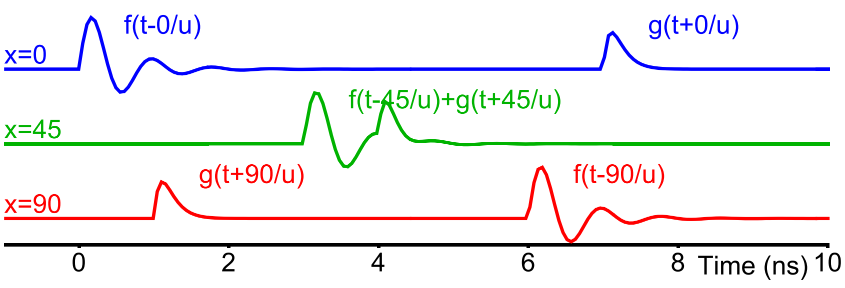

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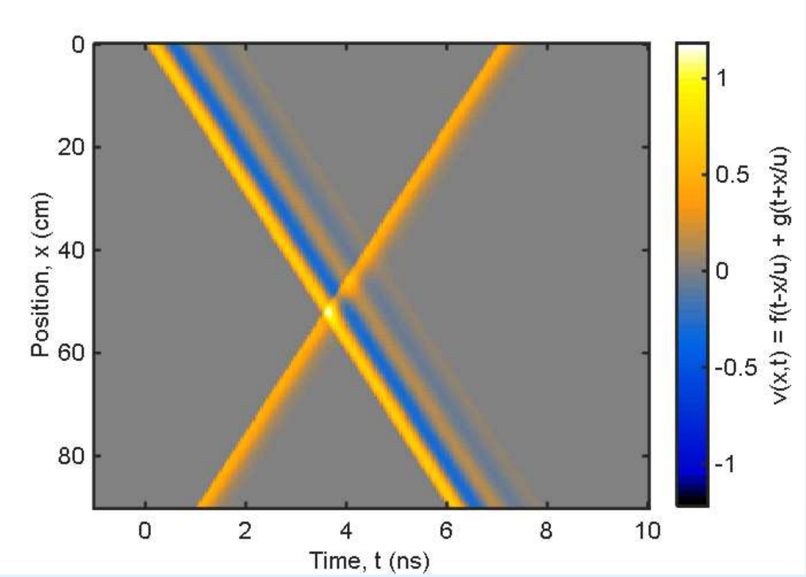
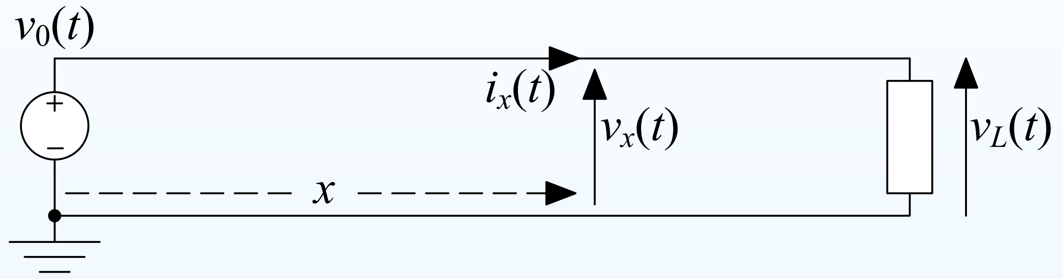
Magically, f and g pass through each other entirely unaltered.



Power Flow

Define $f_x(t) = f\left(t - \frac{x}{u}\right)$ and $g_x(t) = g\left(t + \frac{x}{u}\right)$ to be the forward and backward waveforms at any point, x.



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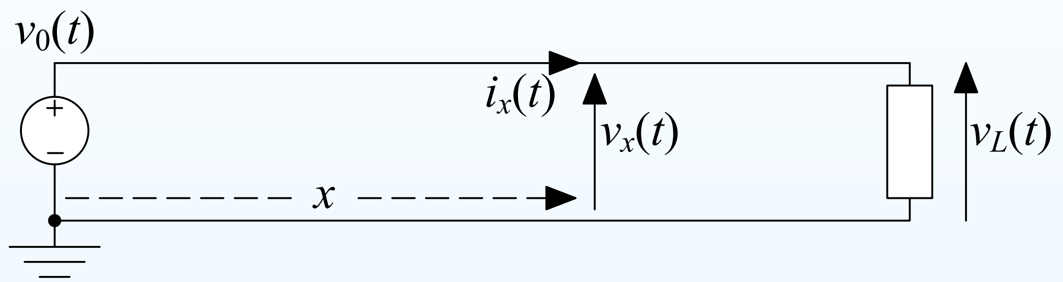

i is always measured in the +ve x direction.

Then $v_x(t) = f_x(t) + g_x(t)$ and $i_x(t) = Z_0^{-1}\left(f_x(t) - g_x(t)\right)$.

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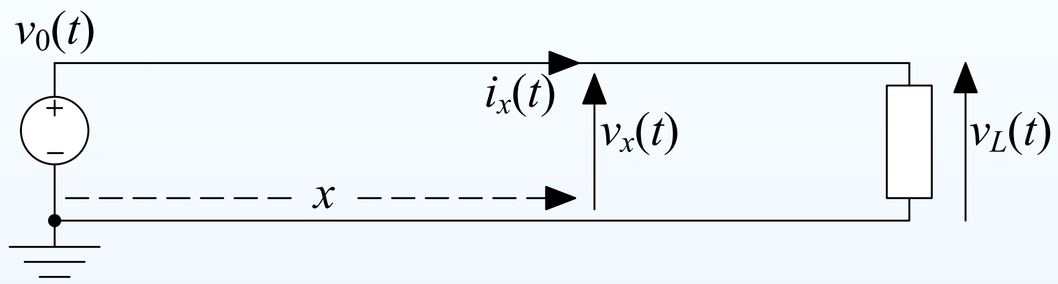

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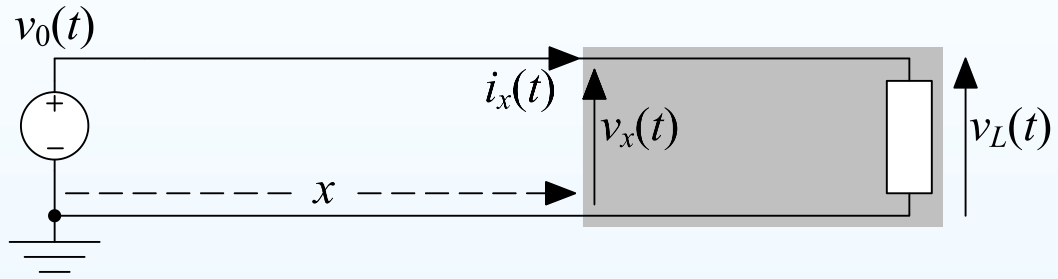

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The power transferred into the shaded region across the boundary at x is $P_x(t) = v_x(t)i_x(t)$

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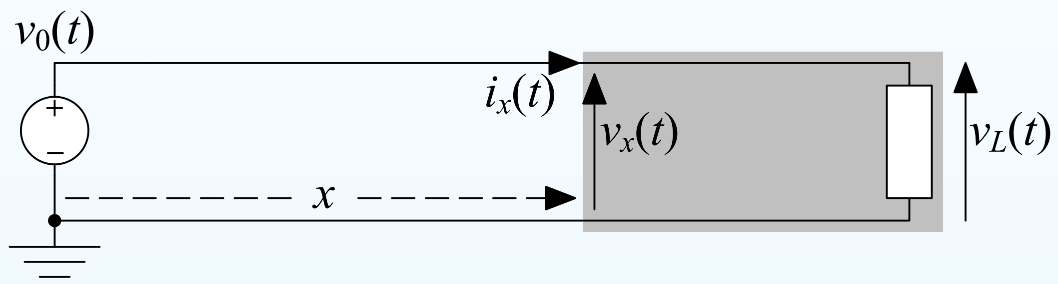

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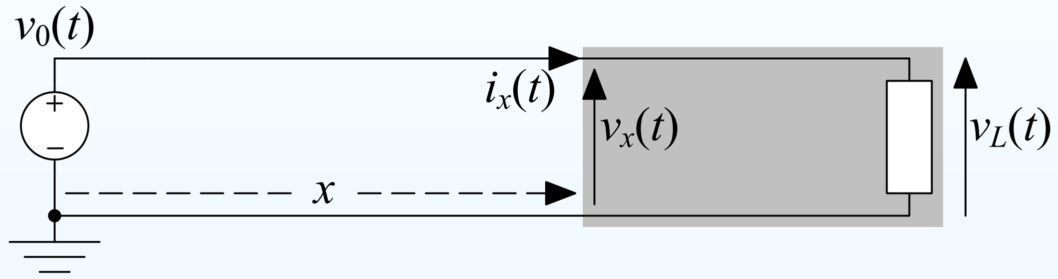

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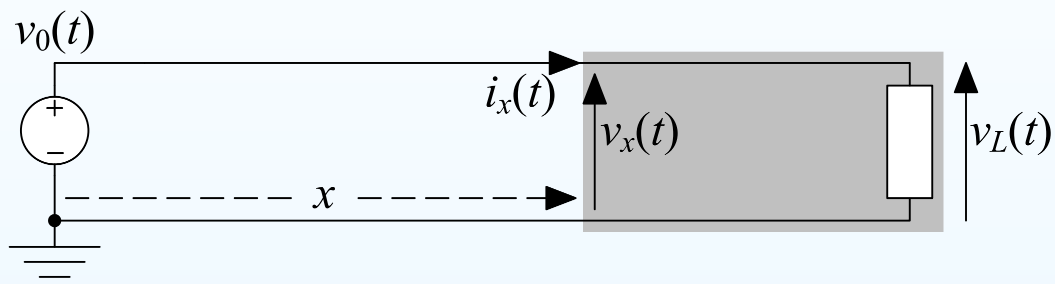

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f_x carries power into shaded area and g_x carries power out independently.

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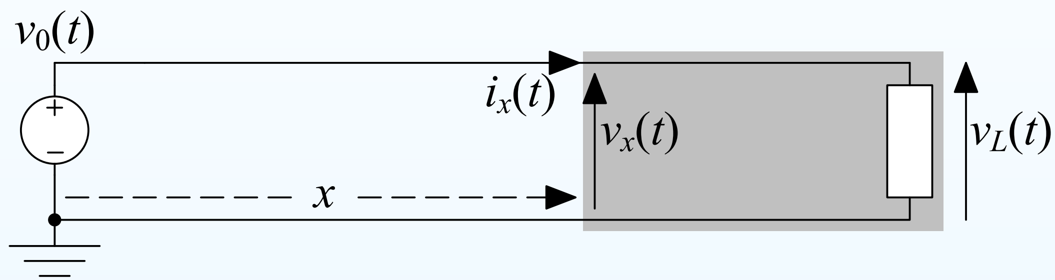

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Power travels in the same direction as the wave.

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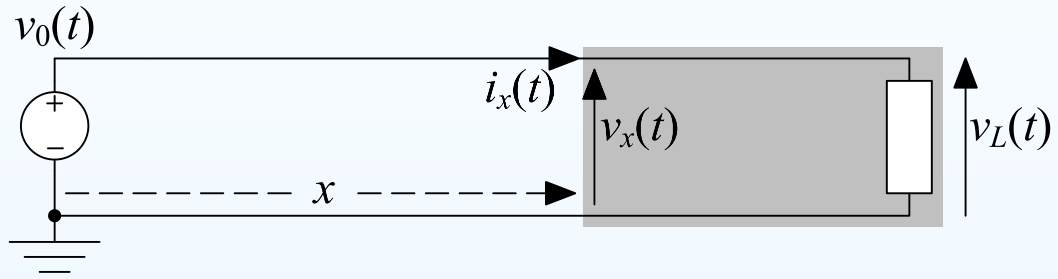

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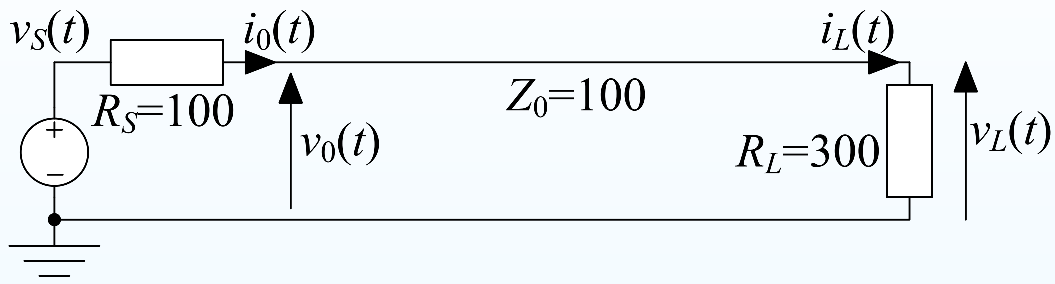
f_x carries power into shaded area and g_x carries power out independently.

Power travels in the same direction as the wave.

The same power as would be absorbed by a [ficticious] resistor of value Z_0.

Reflections



$$v_x = f_x + g_x$$
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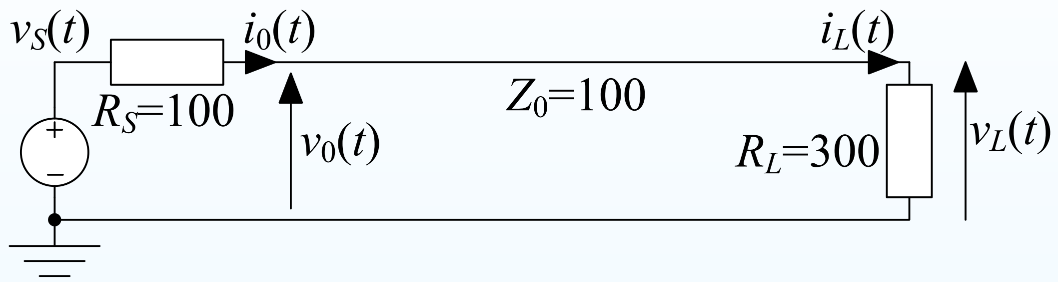
From Ohm's law at $x = L$, we have $v_L(t) = i_L(t)R_L$

Reflections



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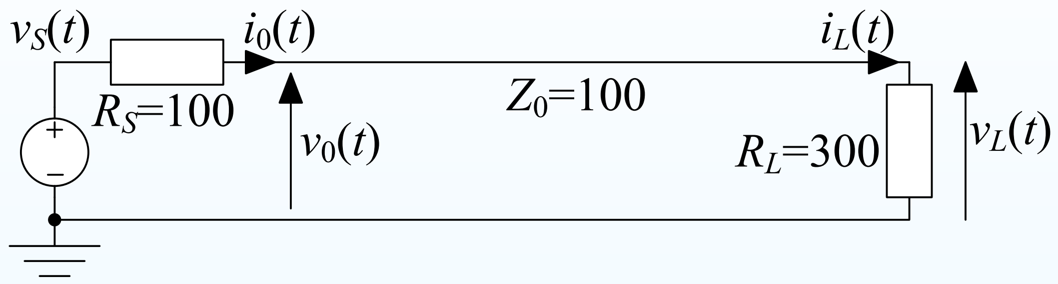

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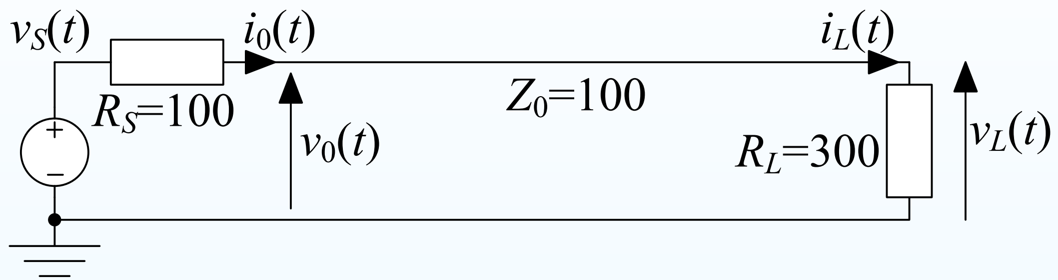
Hence $\left(f_L(t) + g_L(t)\right) = Z_0^{-1}\left(f_L(t) - g_L(t)\right)R_L$

From this: $g_L\left(t\right) = \frac{R_L - Z_0}{R_L + Z_0} \times f_L\left(t\right)$

Reflections



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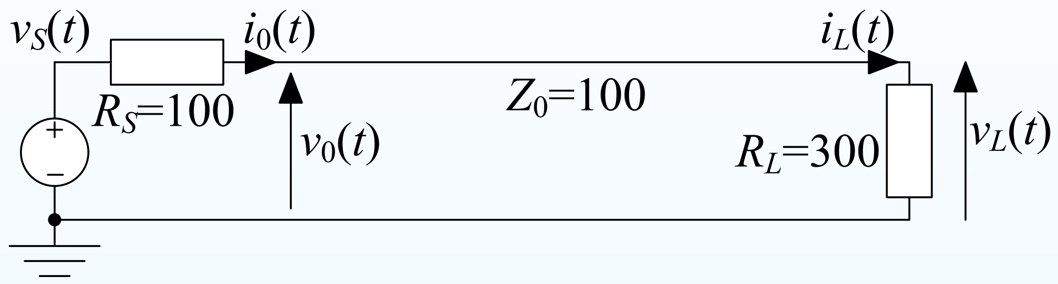
From this: $g_L(t) = \frac{R_L - Z_0}{R_L + Z_0} \times f_L(t)$

We define the *reflection coefficient*: $\rho_L = \frac{g_L(t)}{f_L(t)} = \frac{R_L - Z_0}{R_L + Z_0} = +0.5$

Reflections



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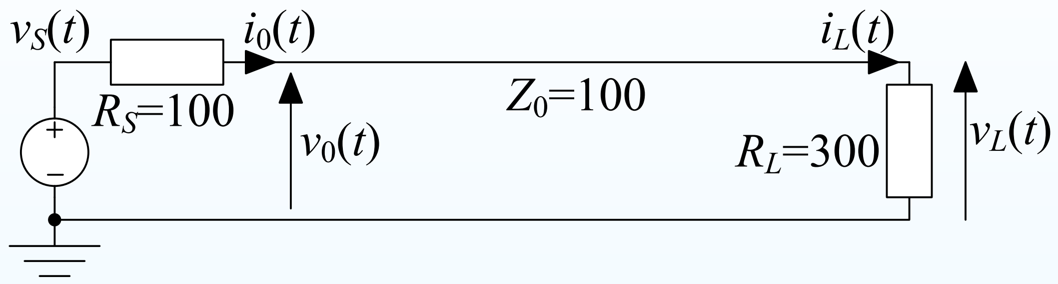
Substituting $g_L(t) = \rho_L f_L(t)$ gives

$v_L(t) = (1 + \rho_L)\,f_L(t)$ and $i_L(t) = (1 - \rho_L)\,Z_0^{-1} f_L(t)$

Reflections



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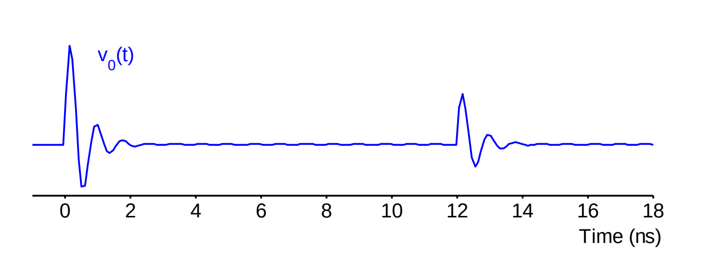
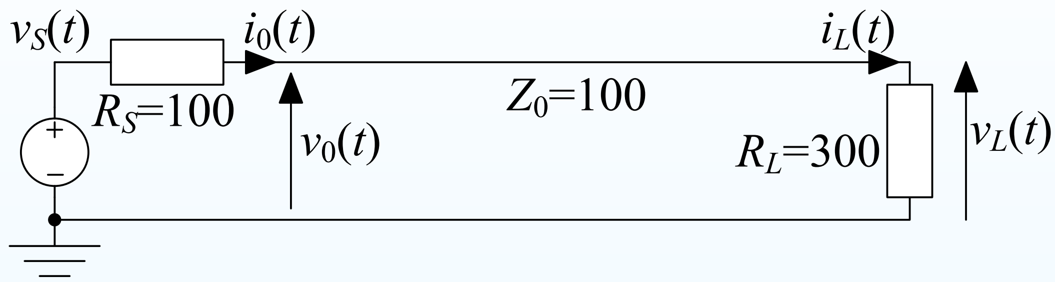


At source end: $g_0(t) = \rho_L f_0\left(t - \frac{2L}{u}\right)$ i.e. delayed by $\frac{2L}{u} = 12$ ns.

Reflections



$$v_x = f_x + g_x$$
$$i_x = Z_0^{-1}(f_x - g_x)$$

From Ohm's law at $x = L$, we have $v_L(t) = i_L(t) R_L$

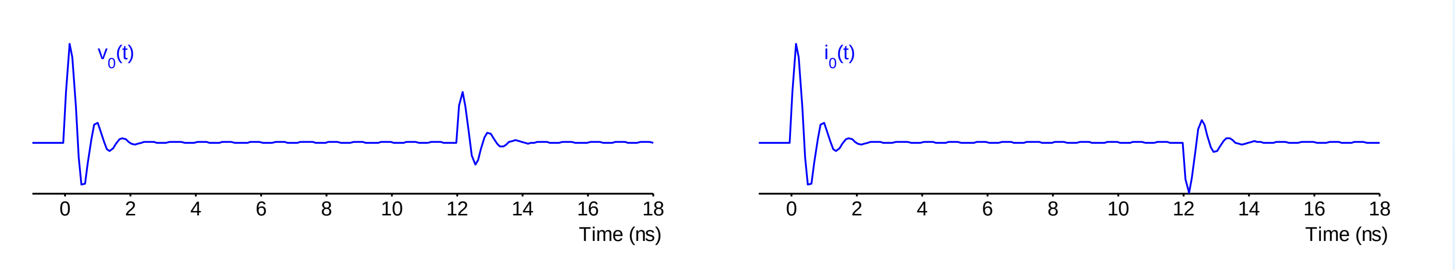
Hence $(f_L(t) + g_L(t)) = Z_0^{-1}(f_L(t) - g_L(t)) R_L$

From this: $g_L(t) = \frac{R_L - Z_0}{R_L + Z_0} \times f_L(t)$

We define the *reflection coefficient*: $\rho_L = \frac{g_L(t)}{f_L(t)} = \frac{R_L - Z_0}{R_L + Z_0} = +0.5$

Substituting $g_L(t) = \rho_L f_L(t)$ gives

$v_L(t) = (1 + \rho_L) f_L(t)$ and $i_L(t) = (1 - \rho_L) Z_0^{-1} f_L(t)$

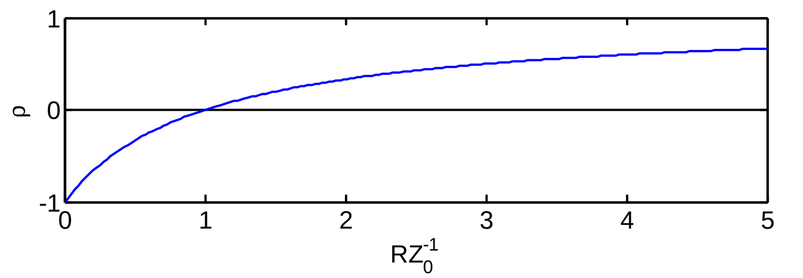


At source end: $g_0(t) = \rho_L f_0\left(t - \frac{2L}{u}\right)$ i.e. delayed by $\frac{2L}{u} = 12$ ns.

Note that the reflected current has been multiplied by $-\rho$.

Reflection Coefficients

$$\rho = \frac{R - Z_0}{R + Z_0} = \frac{\frac{R}{Z_0} - 1}{\frac{R}{Z_0} + 1}$$



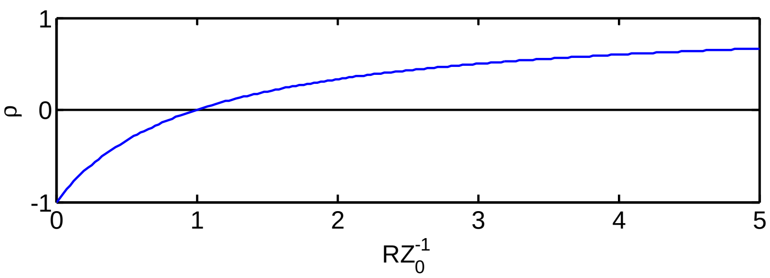
ρ depends on the ratio $\frac{R}{Z_0}$.

$\frac{R}{Z_0}$	ρ	$\frac{v_L(t)}{f(t)}$	$\frac{i_L(t)Z_0}{f(t)}$	Comment
3	$+0.5$			

Reflection Coefficients

$$\rho = \frac{R-Z_0}{R+Z_0} = \frac{\frac{R}{Z_0}-1}{\frac{R}{Z_0}+1}$$

$$\frac{v_L(t)}{f(t)} = 1+\rho$$



ρ depends on the ratio $\frac{R}{Z_0}$.

$\frac{R}{Z_0}$	ρ	$\frac{v_L(t)}{f(t)}$	$\frac{i_L(t)Z_0}{f(t)}$	Comment
3	$+0.5$	1.5		

Reflection Coefficients

$$\rho = \frac{R-Z_0}{R+Z_0} = \frac{\frac{R}{Z_0}-1}{\frac{R}{Z_0}+1}$$

$$\frac{v_L(t)}{f(t)} = 1+\rho$$

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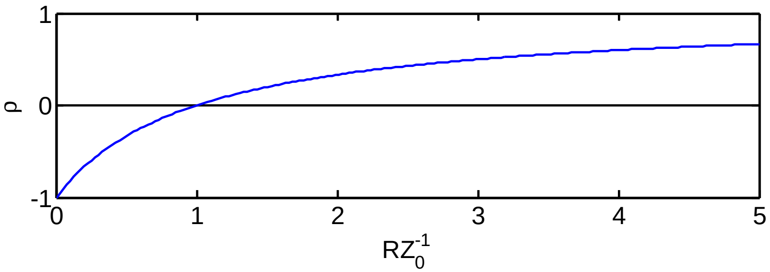

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Reflection Coefficients

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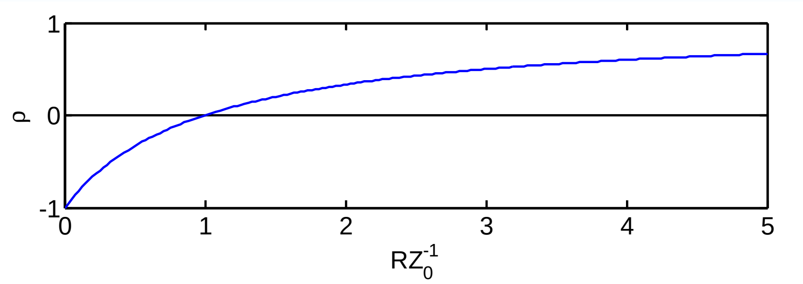

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$\frac{R}{Z_0}$	ρ	$\frac{v_L(t)}{f(t)}$	$\frac{i_L(t)Z_0}{f(t)}$	Comment
3	$+0.5$	1.5	0.5	$R > Z_0 \Rightarrow \rho > 0$

Reflection Coefficients

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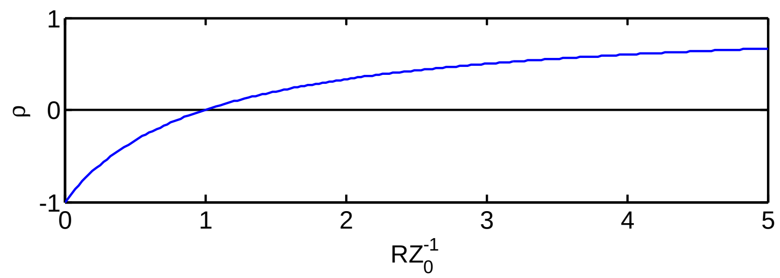

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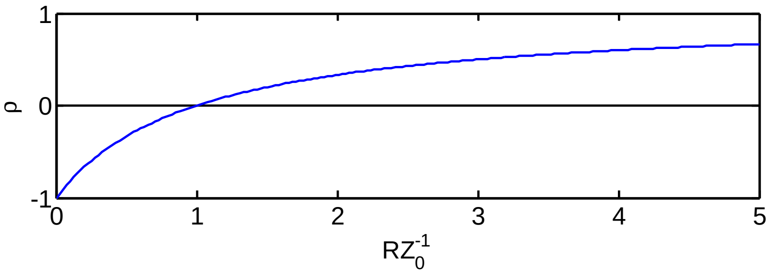

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1	0	1	1	Matched: No reflection at all
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Reflection Coefficients

$$\rho = \frac{R - Z_0}{R + Z_0} = \frac{\frac{R}{Z_0} - 1}{\frac{R}{Z_0} + 1}$$

$$\frac{v_L(t)}{f(t)} = 1 + \rho$$

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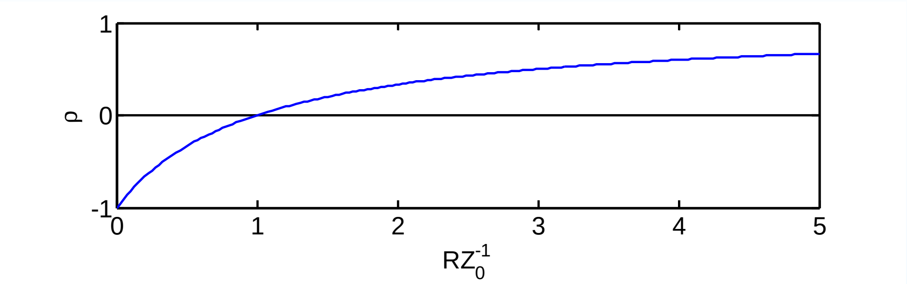

ρ depends on the ratio $\frac{R}{Z_0}$.

$\frac{R}{Z_0}$	ρ	$\frac{v_L(t)}{f(t)}$	$\frac{i_L(t)Z_0}{f(t)}$	Comment
∞	$+1$	2	0	Open circuit: $v_L = 2f$, $i_L \equiv 0$
3	$+0.5$	1.5	0.5	$R > Z_0 \Rightarrow \rho > 0$
1	0	1	1	Matched: No reflection at all
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Reflection Coefficients

$$\rho = \frac{R-Z_0}{R+Z_0} = \frac{\frac{R}{Z_0}-1}{\frac{R}{Z_0}+1}$$

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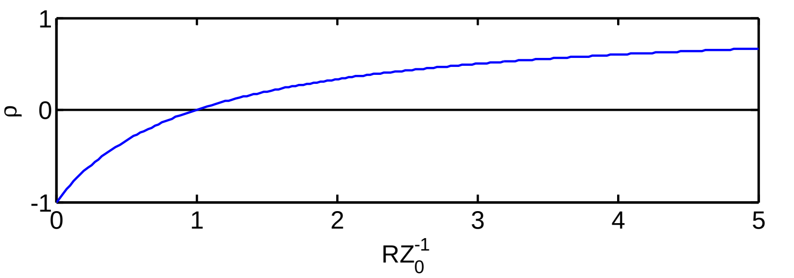

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Reflection Coefficients

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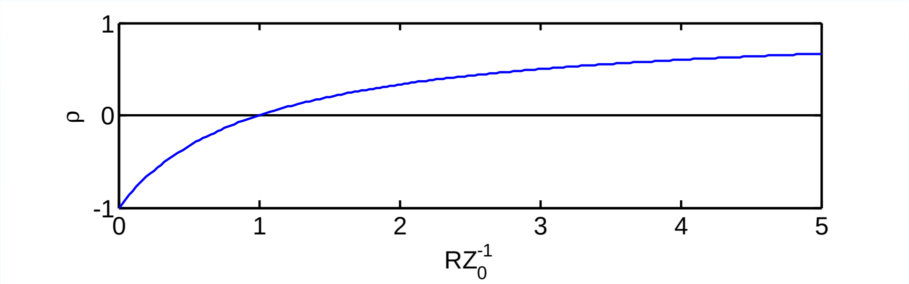

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Note: Reverse mapping is $R = \frac{v_L}{i_L} = \frac{1+\rho}{1-\rho} \times Z_0$

Reflection Coefficients

$$\rho = \frac{R-Z_0}{R+Z_0} = \frac{\frac{R}{Z_0}-1}{\frac{R}{Z_0}+1}$$

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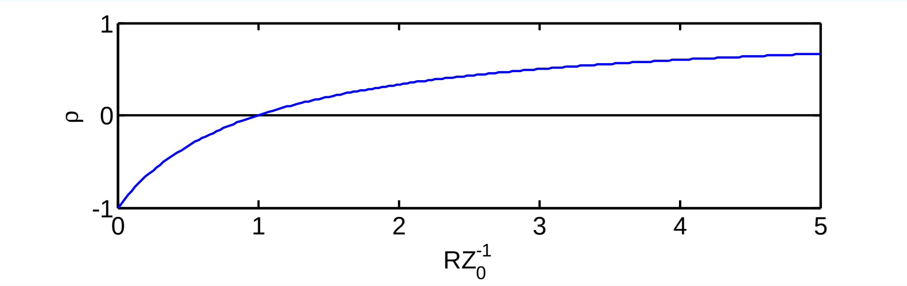

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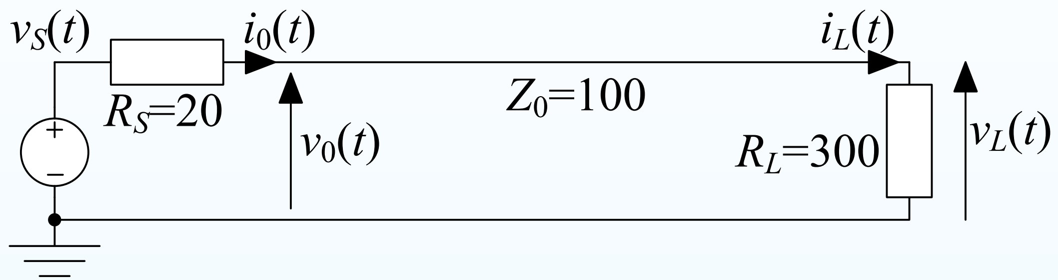
Note: Reverse mapping is $R = \frac{v_L}{i_L} = \frac{1+\rho}{1-\rho} \times Z_0$

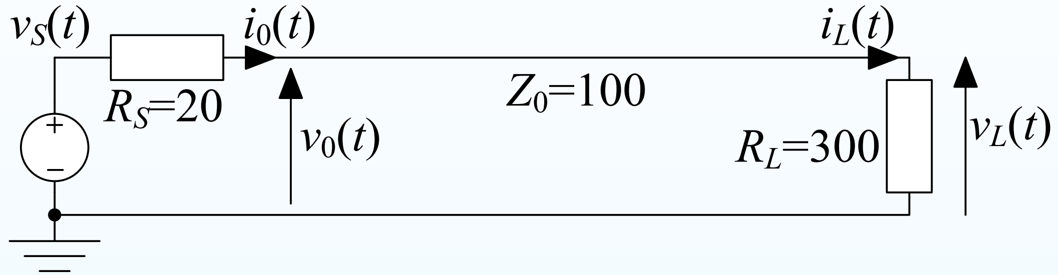
Remember: $\rho \in \{-1, +1\}$ and increases with R.

Driving a line



From Ohm's law at $x = 0$, we have $v_0(t) = v_S(t) - i_0(t)R_S$ where R_S is the Thévenin resistance of the voltage source.

Driving a line



$v_x = f_x + g_x$

$i_x = \frac{f_x - g_x}{Z_0}$

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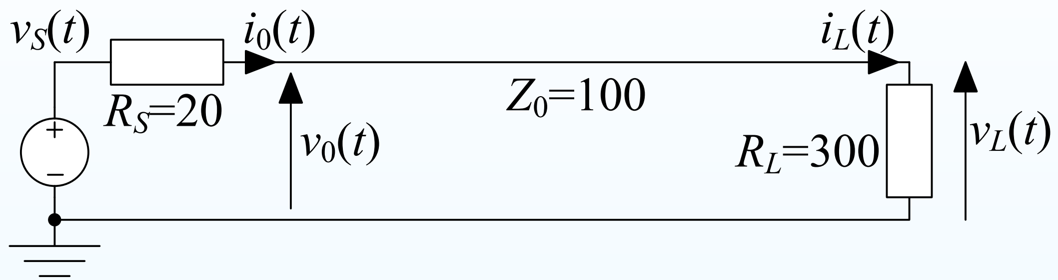
Substituting $v_0(t) = f_0 + g_0$ and $i_0(t) = \frac{f_0 - g_0}{Z_0}$ leads to:

$$f_0(t) = \frac{Z_0}{R_S + Z_0} v_S(t) + \frac{R_S - Z_0}{R_S + Z_0} g_0(t)$$

Driving a line



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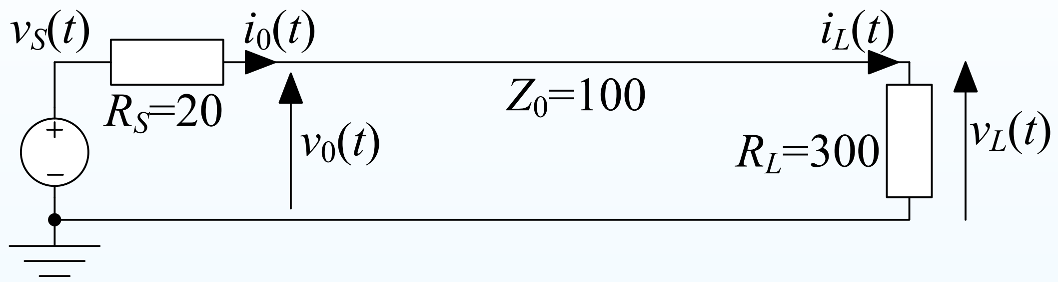
$$f_0(t) = \frac{Z_0}{R_S + Z_0} v_S(t) + \frac{R_S - Z_0}{R_S + Z_0} g_0(t) \triangleq \tau_0 v_S(t) + \rho_0 g_0(t)$$

Driving a line



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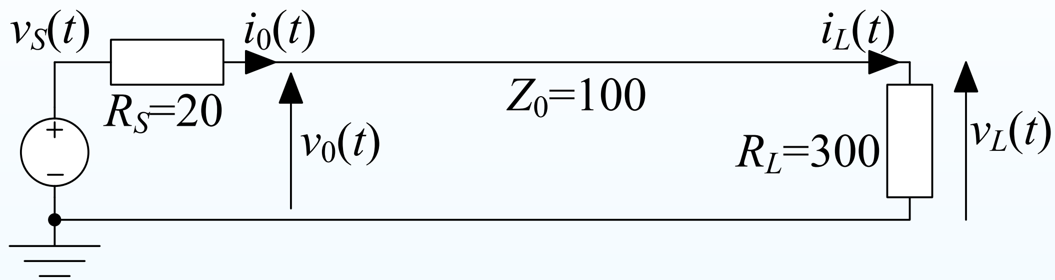
Substituting $v_0(t) = f_0 + g_0$ and $i_0(t) = \frac{f_0 - g_0}{Z_0}$ leads to:

$$f_0(t) = \frac{Z_0}{R_S + Z_0} v_S(t) + \frac{R_S - Z_0}{R_S + Z_0} g_0(t) \triangleq \tau_0 v_S(t) + \rho_0 g_0(t)$$

So $f_0(t)$ is the superposition of two terms:

(1) Input $v_S(t)$ multiplied by $\tau_0 = \frac{Z_0}{R_S + Z_0}$ which is the same as a potential divider if you replace the line with a [ficticious] resistor Z_0.

Driving a line



$$v_x = f_x + g_x$$
$$i_x = \frac{f_x - g_x}{Z_0}$$

From Ohm's law at $x = 0$, we have $v_0(t) = v_S(t) - i_0(t)R_S$ where R_S is the Thévenin resistance of the voltage source.

Substituting $v_0(t) = f_0 + g_0$ and $i_0(t) = \frac{f_0 - g_0}{Z_0}$ leads to:

$$f_0(t) = \frac{Z_0}{R_S + Z_0} v_S(t) + \frac{R_S - Z_0}{R_S + Z_0} g_0(t) \triangleq \tau_0 v_S(t) + \rho_0 g_0(t)$$

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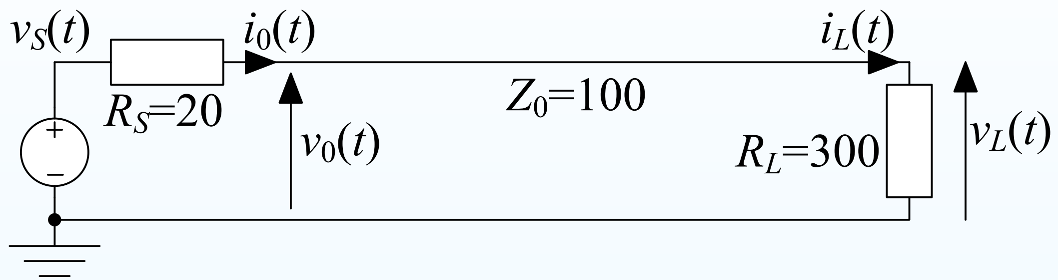
(2) The incoming backward wave, $g_0(t)$, multiplied by a reflection coefficient: $\rho_0 = \frac{R_S - Z_0}{R_S + Z_0}$.

Driving a line



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$$f_0(t) = \frac{Z_0}{R_S + Z_0} v_S(t) + \frac{R_S - Z_0}{R_S + Z_0} g_0(t) \triangleq \tau_0 v_S(t) + \rho_0 g_0(t)$$

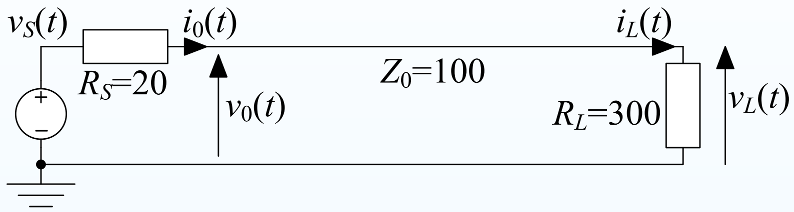
So $f_0(t)$ is the superposition of two terms:

(1) Input $v_S(t)$ multiplied by $\tau_0 = \frac{Z_0}{R_S + Z_0}$ which is the same as a potential divider if you replace the line with a [ficticious] resistor Z_0.

(2) The incoming backward wave, $g_0(t)$, multiplied by a reflection coefficient: $\rho_0 = \frac{R_S - Z_0}{R_S + Z_0}$.

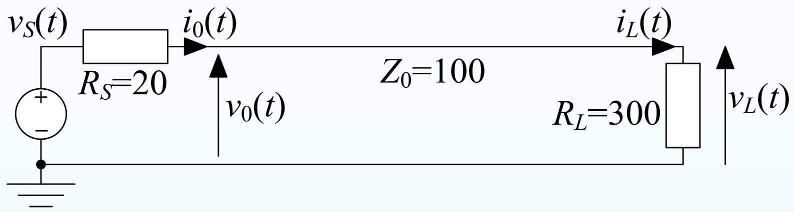
For $R_S = 20$: $\tau_0 = \frac{100}{20+100} = 0.83$ and $\rho_0 = \frac{20-100}{20+100} = -0.67$.

Multiple Reflections



$$\rho_0 = -\frac{2}{3}$$
$$\rho_L = \frac{1}{2}$$
$$v_x = f_x + g_x$$

Multiple Reflections



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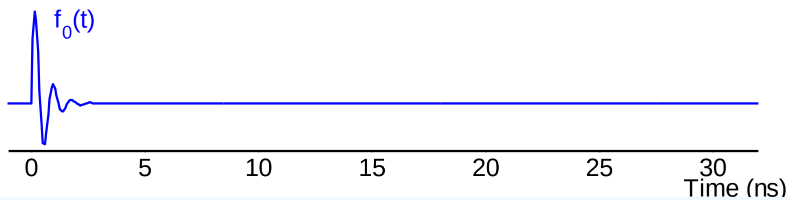

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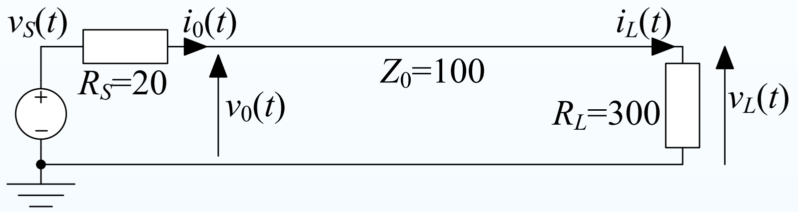

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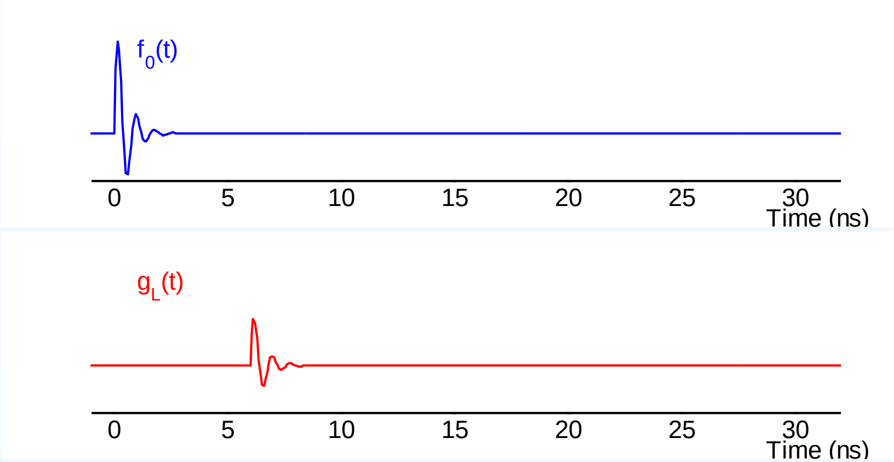

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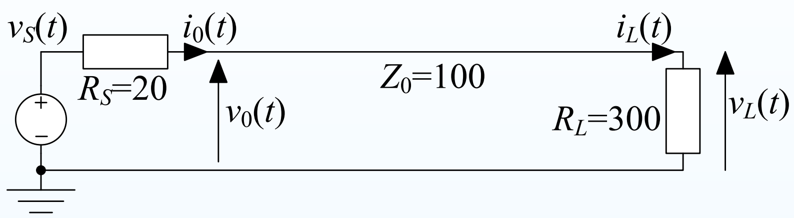

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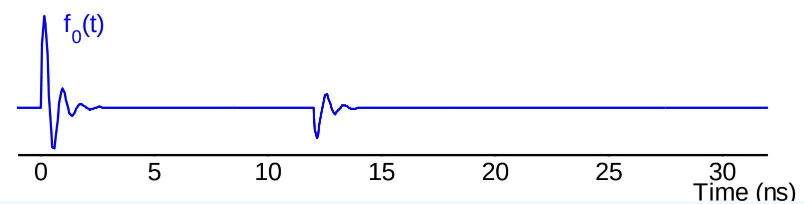

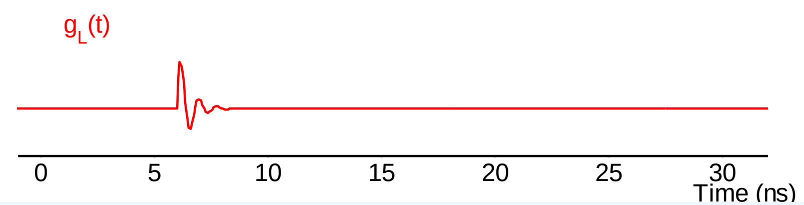

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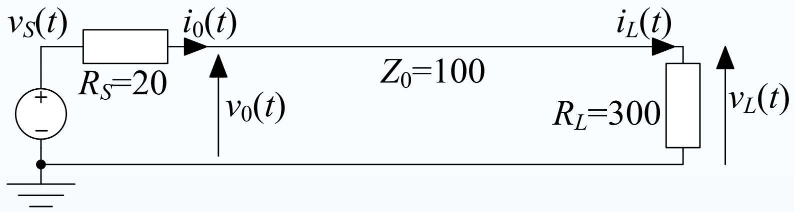

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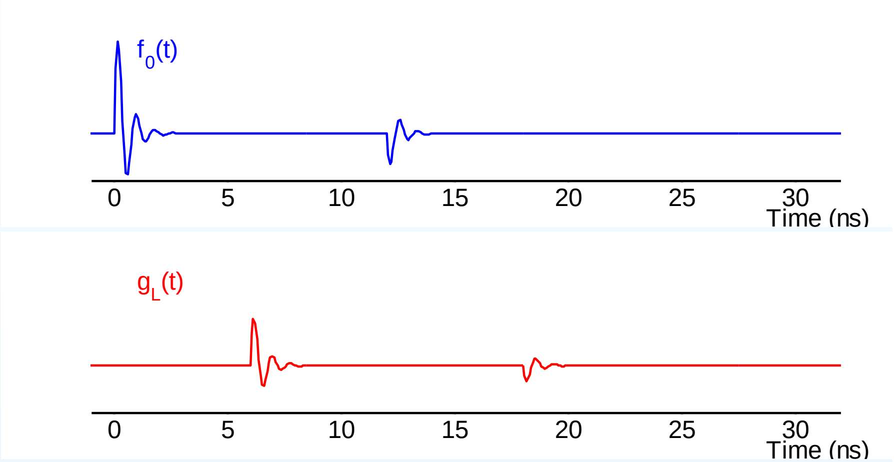

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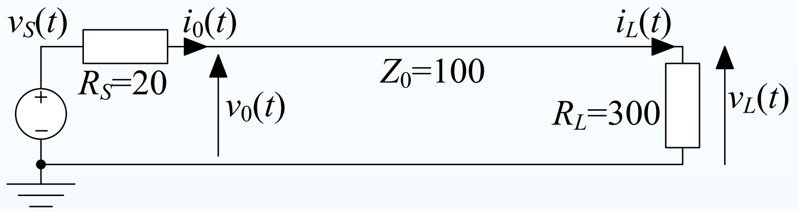

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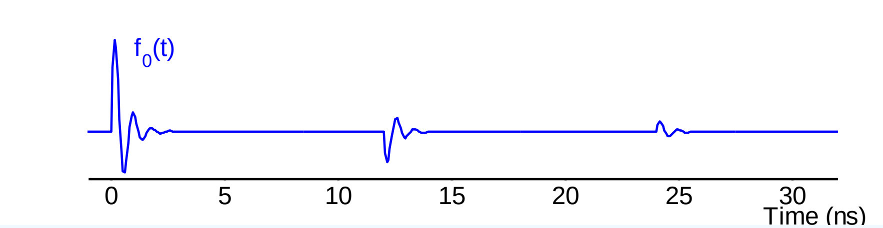

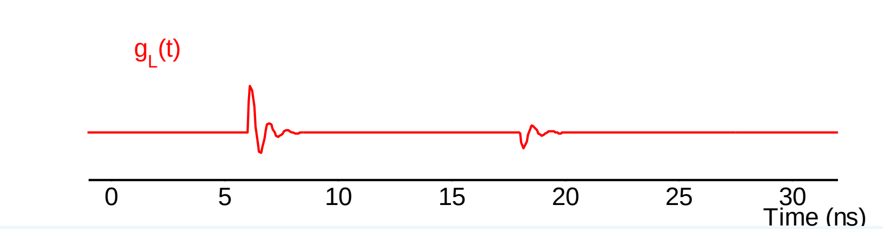

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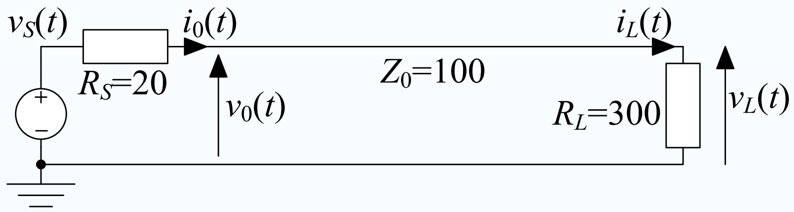

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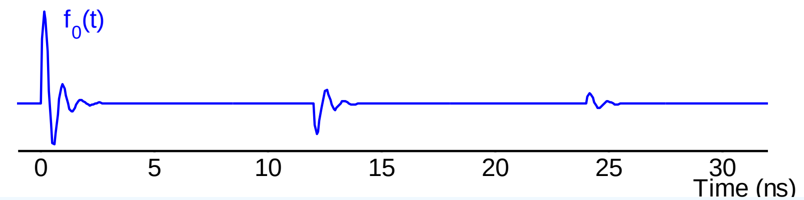

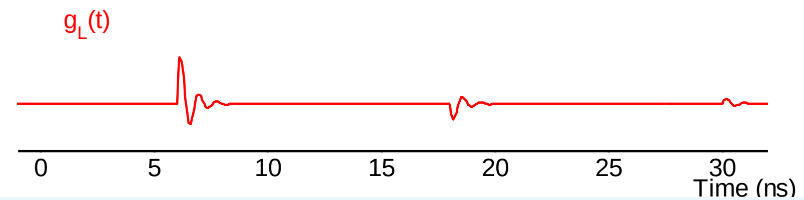

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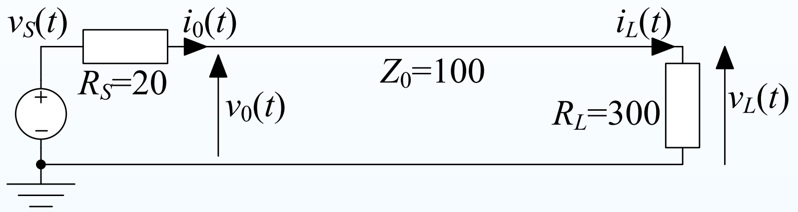

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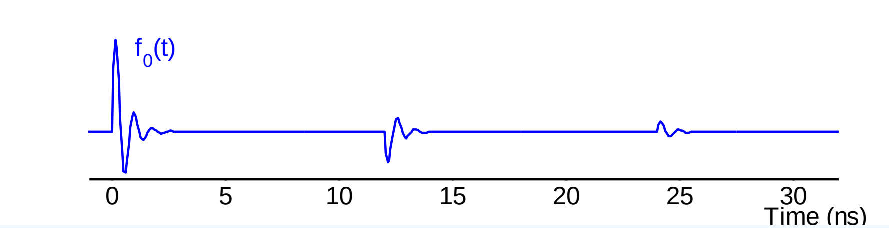

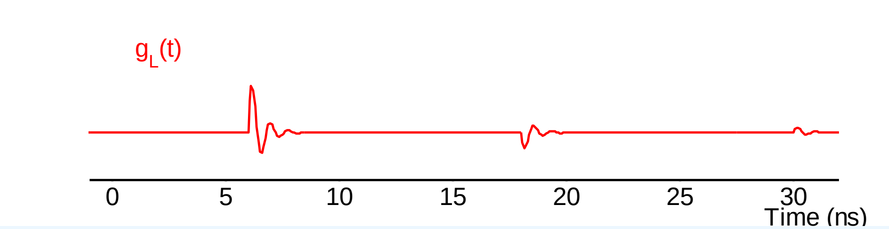

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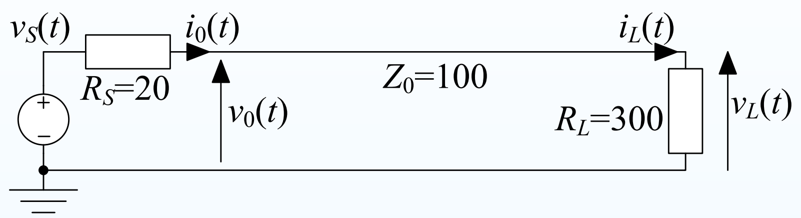

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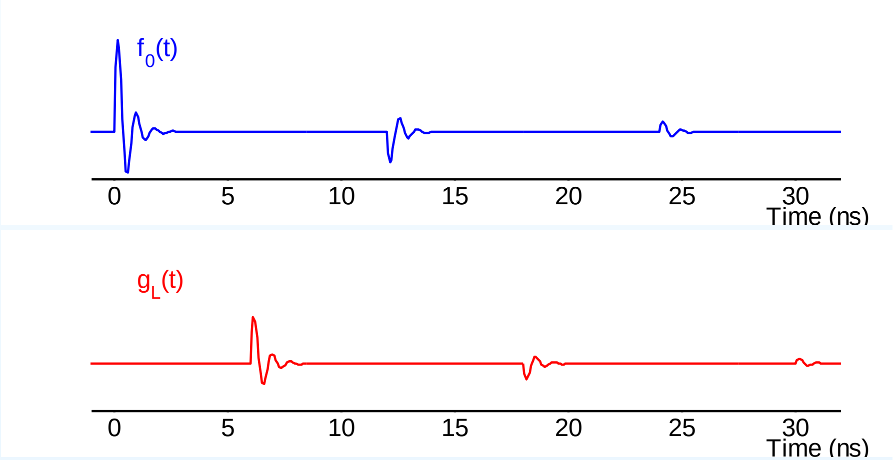

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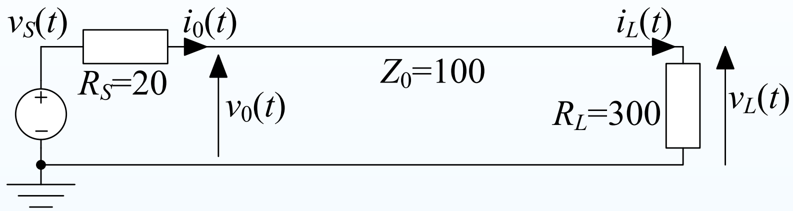

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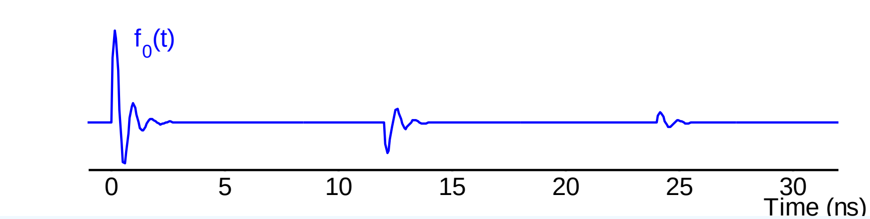

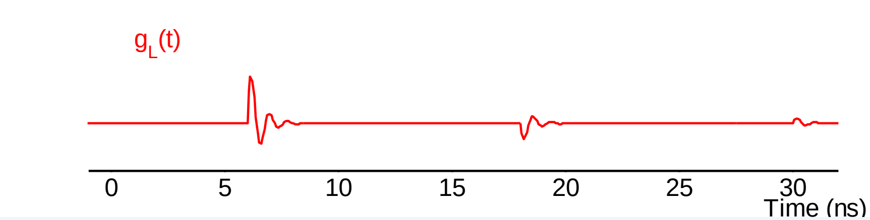

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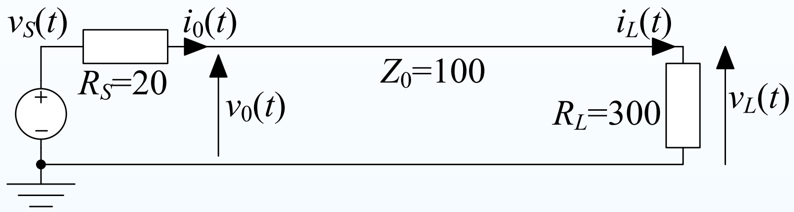

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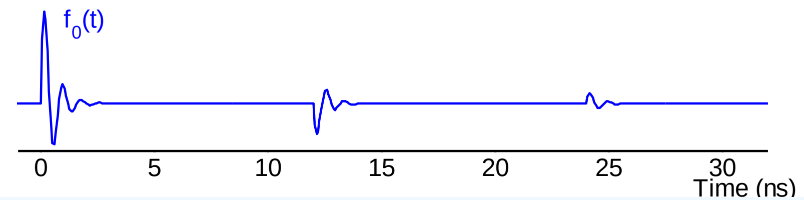

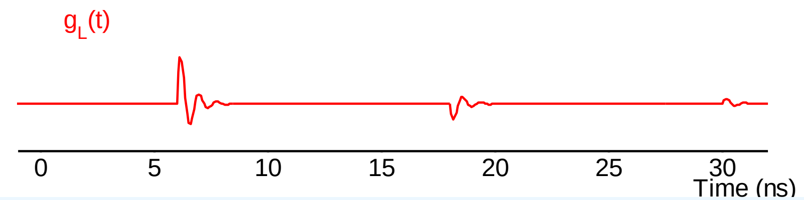

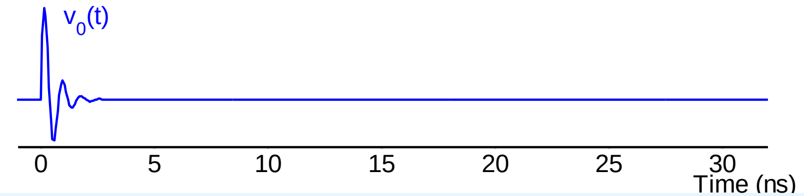

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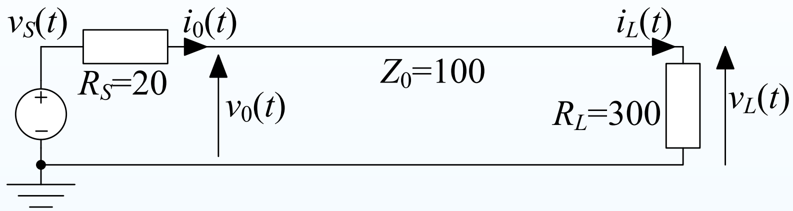

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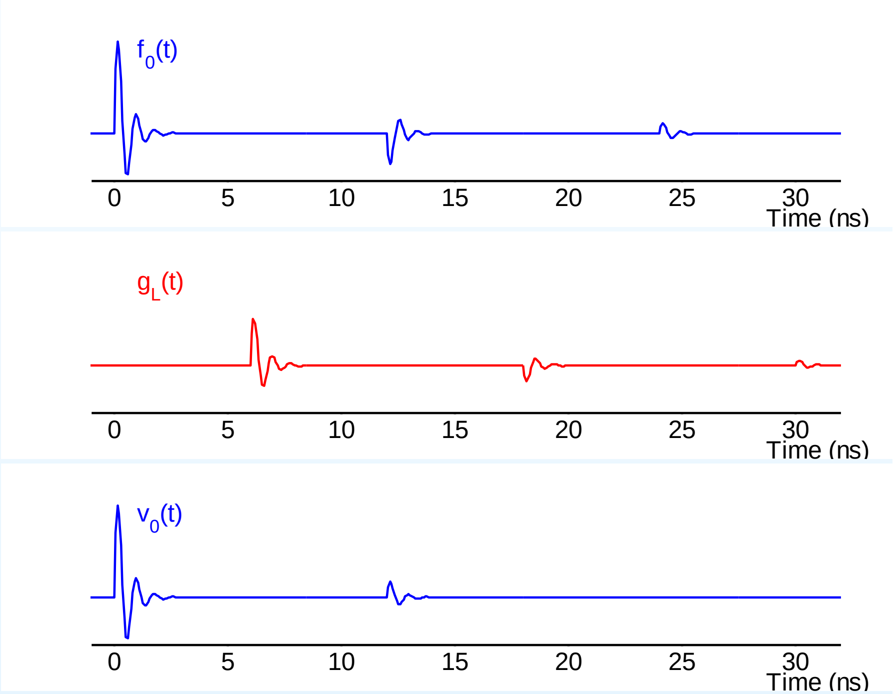

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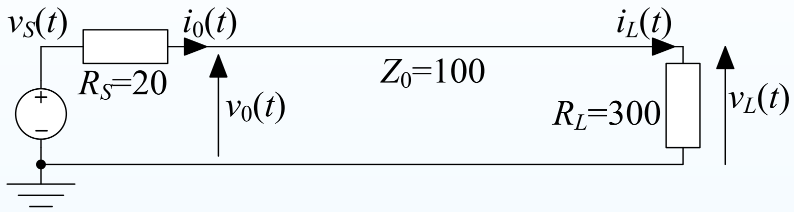

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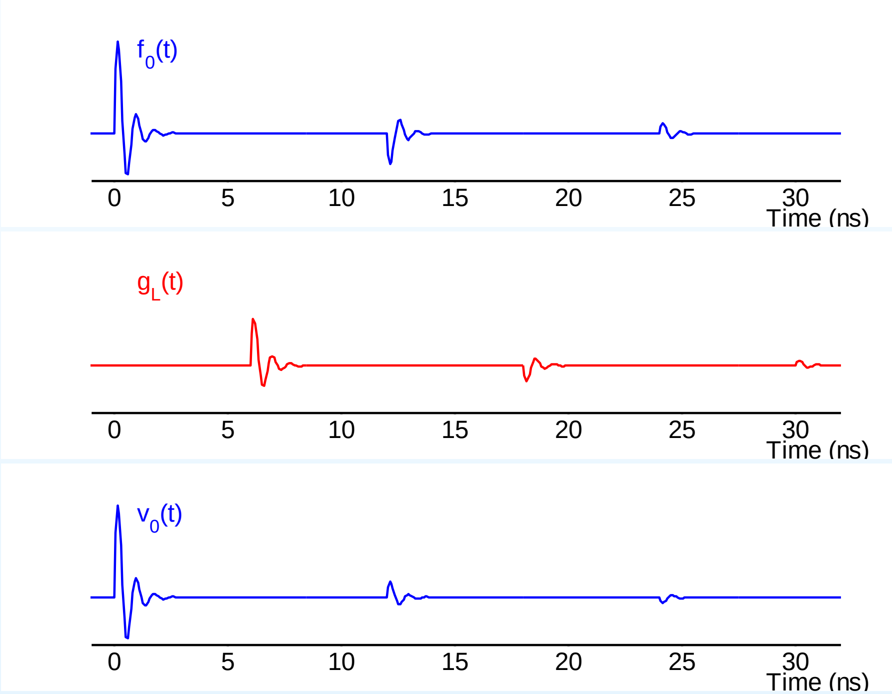

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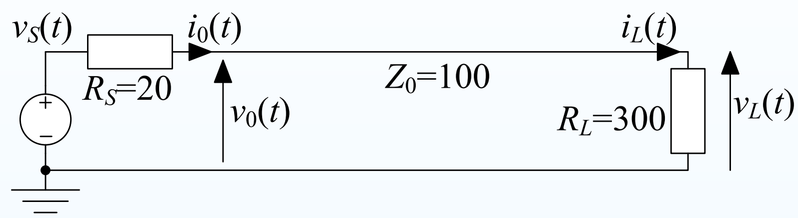

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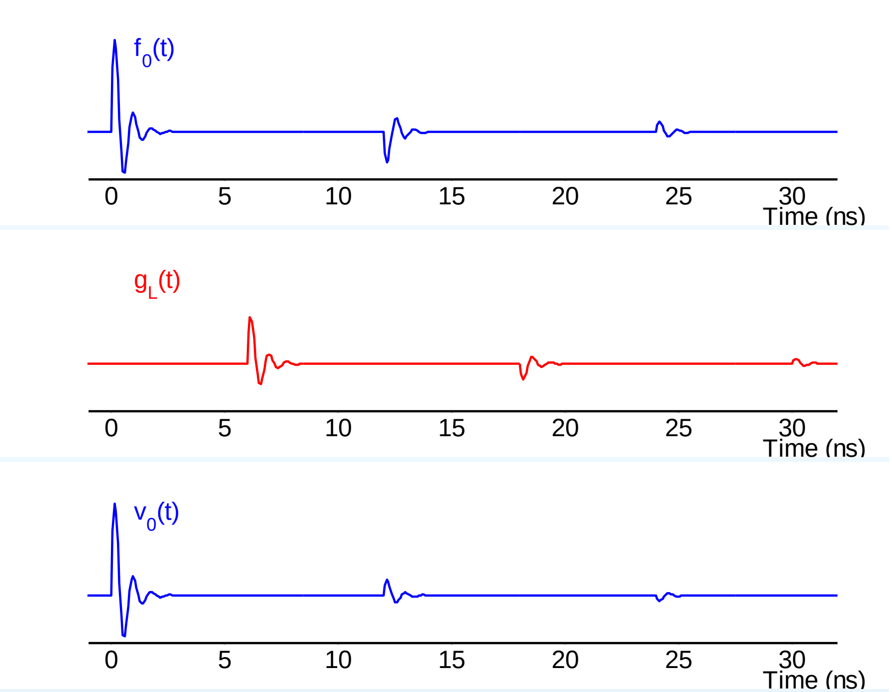

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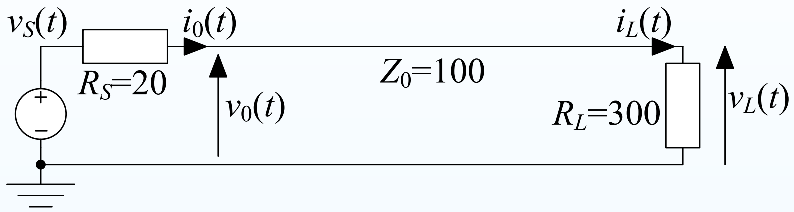

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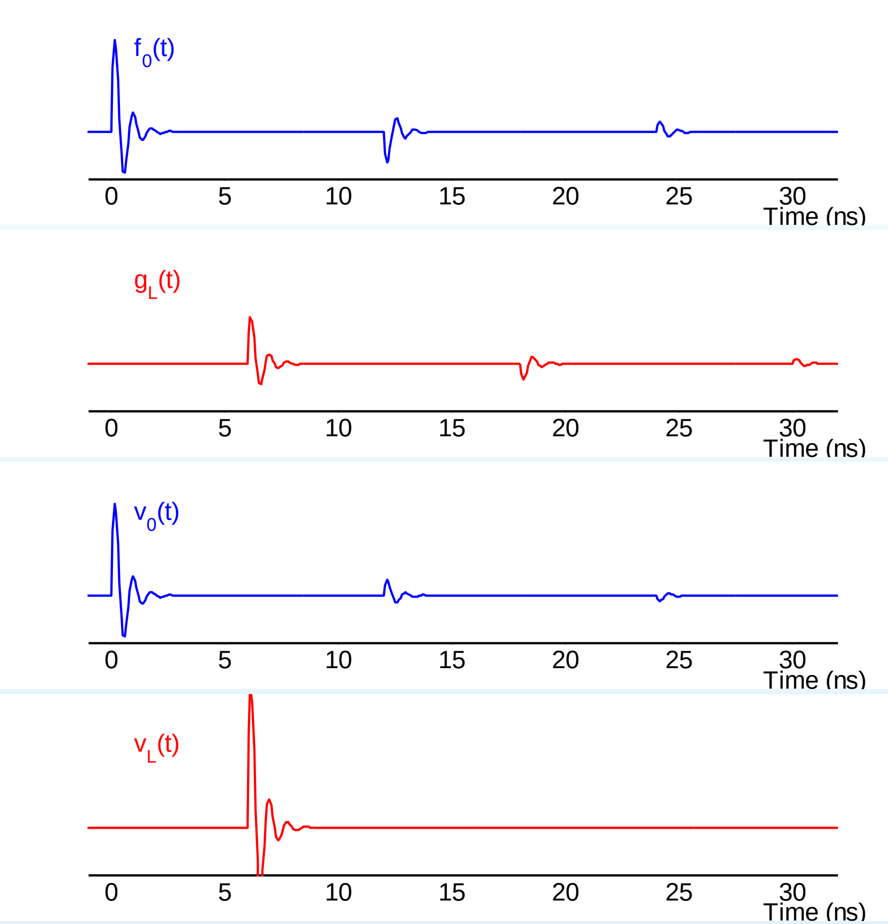

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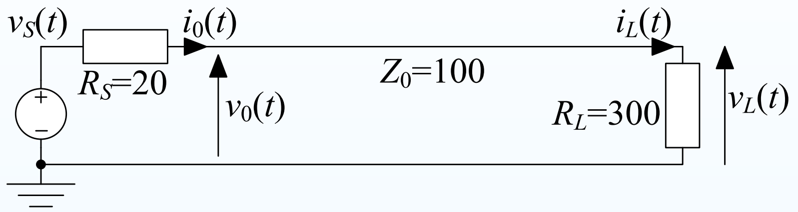

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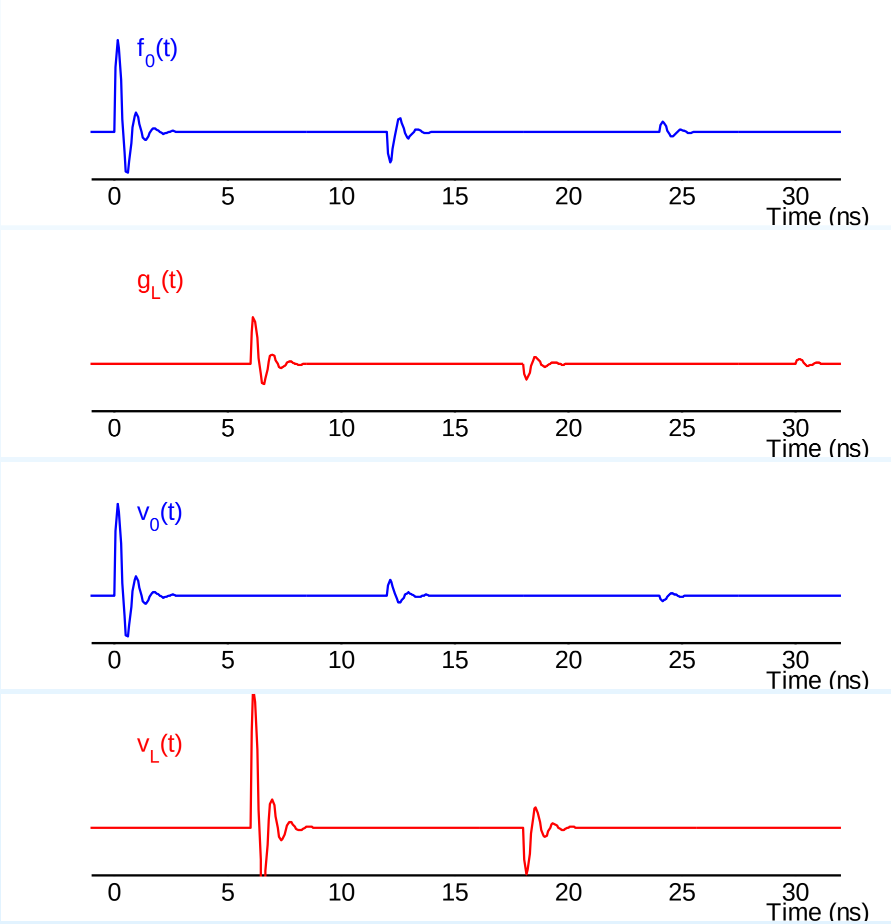

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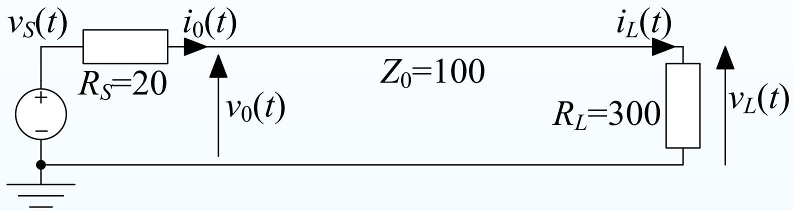

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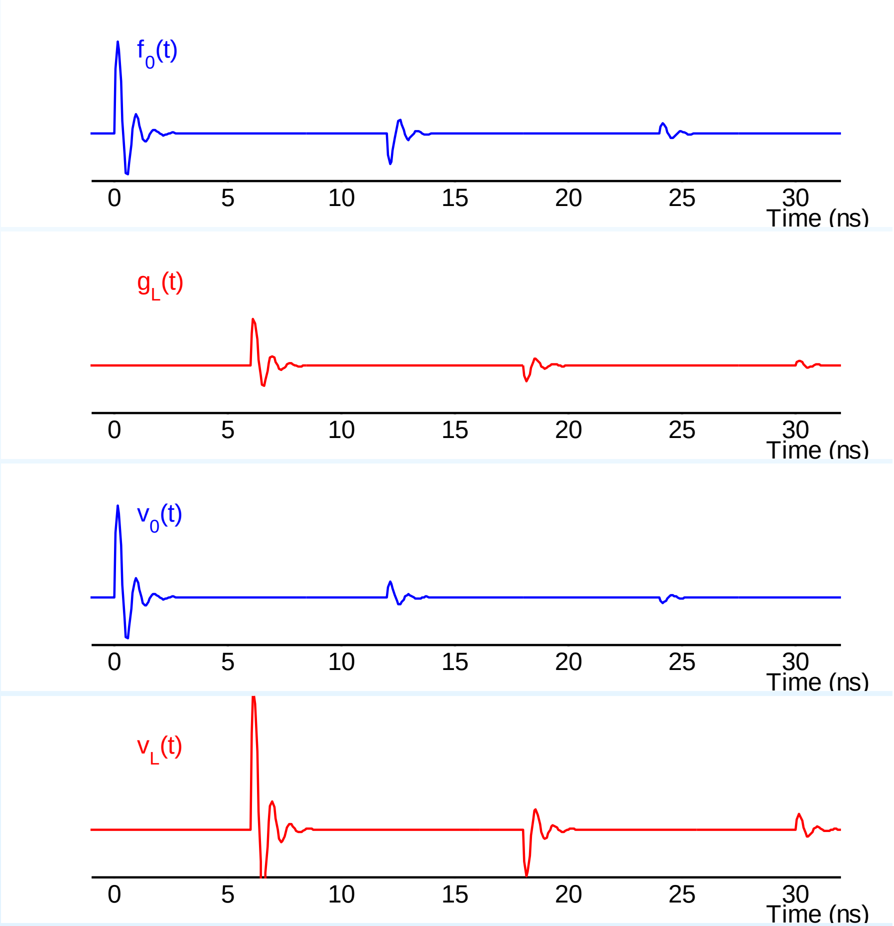
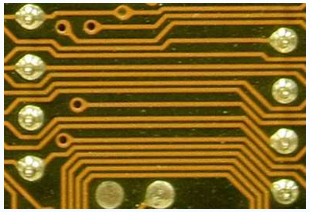


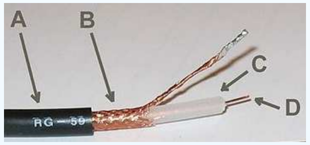
Transmission Line Characteristics

Integrated circuits & Printed circuit boards

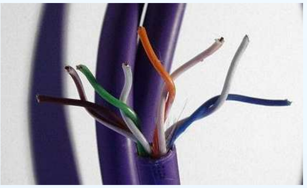


High speed digital or high frequency analog interconnections
$Z_0 \approx 100\,\Omega$, $u \approx 15$ cm/ns.

Long Cables



Coaxial cable ("coax"): unaffacted by external fields; use for antennae and instrumentation.
$Z_0 = 50$ or $75\,\Omega$, $u \approx 25$ cm/ns.



Twisted Pairs: cheaper and thinner than coax and resistant to magnetic fields; use for computer network and telephone cabling. $Z_0 \approx 100\,\Omega$, $u \approx 19$ cm/ns.

When do you have to bother?

Answer: long cables or high frequencies. You can completely ignore transmission line effects if $\text{length} \ll \frac{u}{\text{frequency}} = \text{wavelength}$.

- Audio (< 20 kHz) never matters.
- Computers (1 GHz) usually matters.
- Radio/TV usually matters.

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  - f is infinite sum of copies of the input signal delayed successively by the round-trip delay, $\frac{2L}{u}$, and multiplied by $\rho_L \rho_0$.